

# Insights into Standard Model from Quantum Gravity

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# INTRODUCTION

**A short recap of the spinfoam quantization method:**

- Step 1: Rewrite the GR action — as a topological  $BF$  theory plus simplicity constraint,

$$S_{\text{Plebanski}}[B, \omega, \phi] = \int_{\mathcal{M}_4} \langle B \wedge F(\omega) \rangle_{\mathfrak{g}} + \langle \phi(B \wedge B) \rangle_{\mathfrak{g}},$$

where the Lie group  $G$  is Lorentz-like, and  $\mathfrak{g}$  is its Lie algebra.

- Step 2: Quantize the topological sector — a state sum over a triangulated manifold  $T(\mathcal{M}_4)$ ,

$$Z_{BF} = \sum_{\Lambda} \prod_v \mathcal{A}_v(\Lambda) \prod_e \mathcal{A}_e(\Lambda) \prod_{\Delta} \mathcal{A}_{\Delta}(\Lambda) \prod_{\tau} \mathcal{A}_{\tau}(\Lambda) \prod_{\sigma} \mathcal{A}_{\sigma}(\Lambda).$$

“Colors”  $\Lambda$  are reps of  $G$ , amplitudes  $\mathcal{A}$  chosen so that  $Z_{BF}$  is invariant wrt. Pachner moves.

- Step 3: Impose the simplicity constraint — deform the invariant  $Z_{BF}$  by modifying the amplitudes and reps,

$$Z_{BF} \rightarrow Z_{GR} : \quad \mathcal{A}(\Lambda) \rightarrow W(j), \quad j = f(\Lambda),$$

obtaining a state sum  $Z_{GR}$  which defines a spinfoam model (Barret-Crane, EPRL/FK, etc).

***Key question — how to add matter fields into the above?***

# HIGHER CATEGORY THEORY

**A flash introduction to higher category theory:**

- An  $n$ -category is a set of *objects* with:
  - *morphisms* (maps between objects),
  - *2-morphisms* (maps between morphisms),
  - *3-morphisms* (maps between 2-morphisms), ... up to  *$n$ -morphisms*,

along with certain axioms to provide suitable rules for composition, associativity, etc.

- An  $n$ -group is a special case of an  $n$ -category, which has only one object, and all morphisms are invertible.

**A more detailed introduction to higher category theory:**

⇒ look up “**An Invitation to Higher Gauge Theory**” [Baez, Huerta (2011)]

**The purpose of  $n$ -groups (for physicists):**

⇒ *more fine-grained description of symmetry* using an  $n$ -group, than using a group,

⇒ generalization of differential geometry: *parallel transport, connection, holonomy, curvature*.

# LIE 3-GROUPS

Focus on a Lie 3-group, specified in detail by a 2-crossed module:

[Faria Martins, Picken (2011); Wang (2014)]

$$( L \xrightarrow{\delta} H \xrightarrow{\partial} G , \triangleright , \{ -, - \} )$$

- $L, H, G$  — Lie groups,
- $\delta, \partial$  — boundary morphisms,
- $\triangleright$  — action of  $G$ ,  $\triangleright : G \times G \rightarrow G$ ,  $\triangleright : G \times H \rightarrow H$ ,  $\triangleright : G \times L \rightarrow L$ ,
- $\{ -, - \}$  — Peiffer lifting,  $\{ -, - \} : H \times H \rightarrow L$ .

**Axioms that hold among these maps:**

Chain complex:	$\partial\delta = 1_G$ ,
Conjugation:	$g \triangleright g_0 = g g_0 g^{-1}$ ,
$G$ -equivariance of $\partial$ and $\delta$ :	$g \triangleright \partial h = \partial(g \triangleright h)$ , $g \triangleright \delta l = \delta(g \triangleright l)$ ,
$G$ -equivariance of lifting:	$g \triangleright \{h_1, h_2\} = \{g \triangleright h_1, g \triangleright h_2\}$ ,
Peiffer commutator:	$\delta \{h_1, h_2\} = h_1 h_2 h_1^{-1} (\partial h_1) \triangleright h_2^{-1}$ ,
$L$ -commutator:	$\{\delta l_1, \delta l_2\} = l_1 l_2 l_1^{-1} l_2^{-1}$ ,
$\delta$ -lifting relation:	$\{\delta l, h\} \{h, \delta l\} = l(\partial h \triangleright l^{-1})$ ,
Left product rule:	$\{h_1 h_2, h_3\} = \{h_1, h_2 h_3 h_2^{-1}\} \partial h_1 \triangleright \{h_2, h_3\}$ .

# LIE 3-GROUPS

Purpose of all this — to generalize the notion of parallel transport, from curves to surfaces to volumes:

- Connection generalized to a 3-connection  $(\alpha, \beta, \gamma)$ , a triple of algebra-valued differential forms:

$$\begin{aligned}\alpha &= \alpha^\alpha{}_\mu(x) \tau_\alpha \otimes \mathbf{d}x^\mu && \in \mathfrak{g} \otimes \Lambda^1(\mathcal{M}), \\ \beta &= \frac{1}{2} \beta^a{}_{\mu\nu}(x) t_a \otimes \mathbf{d}x^\mu \wedge \mathbf{d}x^\nu && \in \mathfrak{h} \otimes \Lambda^2(\mathcal{M}), \\ \gamma &= \frac{1}{3!} \gamma^A{}_{\mu\nu\rho}(x) T_A \otimes \mathbf{d}x^\mu \wedge \mathbf{d}x^\nu \wedge \mathbf{d}x^\rho && \in \mathfrak{l} \otimes \Lambda^3(\mathcal{M}).\end{aligned}$$

- Line holonomy generalized to surface and volume holonomies:

$$g = \mathcal{P}\exp \int_{\mathcal{P}_1} \alpha, \quad h = \mathcal{S}\exp \int_{\mathcal{S}_2} \beta, \quad l = \mathcal{V}\exp \int_{\mathcal{V}_3} \gamma.$$

- Ordinary curvature generalized to 3-curvature  $(\mathcal{F}, \mathcal{G}, \mathcal{H})$ , where:

$$\begin{aligned}\mathcal{F} &= \mathbf{d}\alpha + \alpha \wedge \alpha - \partial\beta, \\ \mathcal{G} &= \mathbf{d}\beta + \alpha \wedge^\triangleright \beta - \delta\gamma, \\ \mathcal{H} &= \mathbf{d}\gamma + \alpha \wedge^\triangleright \gamma - \{\beta \wedge \beta\}.\end{aligned}$$

# HIGHER GAUGE THEORY

At this point one can construct the action for a higher gauge theory:

$$S_{3BF} = \int_{\mathcal{M}_4} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} + \langle C \wedge \mathcal{G} \rangle_{\mathfrak{h}} + \langle D \wedge \mathcal{H} \rangle_{\mathfrak{l}}.$$

$\Rightarrow$  Topological  $3BF$  theory, based on the 3-group  $(L \xrightarrow{\delta} H \xrightarrow{\partial} G, \triangleright, \{-, -\})$ .

The physical interpretation of the Lagrange multipliers  $C$  and  $D$ :

- for  $H = \mathbb{R}^4$ , multiplier  $C$  can be **interpreted as the tetrad 1-form**:

$$C \rightarrow e = e^a{}_{\mu}(x) t_a \otimes dx^{\mu}, \quad [\text{Miković, MV (2012)}]$$

- for given  $L$ , multiplier  $D$  can be **interpreted as the set of matter fields**:

$$D \rightarrow \phi = \phi^A(x) T_A. \quad [\text{Radenković, MV (2019)}]$$

$\Rightarrow$  The action thus becomes:

$$S_{3BF} = \int_{\mathcal{M}_4} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} + \langle e \wedge \mathcal{G} \rangle_{\mathfrak{h}} + \langle \phi \wedge \mathcal{H} \rangle_{\mathfrak{l}}.$$

# THE STANDARD MODEL

How many real-valued field components do we have in the Standard Model?

The fermion sector gives us:

$$\left. \begin{array}{cccc} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L & \begin{pmatrix} u_r \\ d_r \end{pmatrix}_L & \begin{pmatrix} u_g \\ d_g \end{pmatrix}_L & \begin{pmatrix} u_b \\ d_b \end{pmatrix}_L \\ \nu_e)_R & (u_r)_R & (u_g)_R & (u_b)_R \\ (e^-)_R & (d_r)_R & (d_g)_R & (d_b)_R \end{array} \right\} = 16 \frac{\text{spinors}}{\text{family}} \times$$

$$\times 3 \text{ families} \times 4 \frac{\text{real-valued components}}{\text{spinor}} = 192 \text{ real-valued components } \phi^A.$$

The Higgs sector gives us:

$$\left. \begin{array}{c} \phi^+ \\ \phi_0 \end{array} \right\} = 2 \text{ complex scalar fields} = 4 \text{ real-valued components } \phi^A.$$

This suggests the structure for  $L$  in the form:

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}),$$

where  $\mathbb{G}$  is the Grassmann algebra.

# THE STANDARD MODEL

The actions  $\triangleright : G \times L \rightarrow L$  and  $\triangleright : G \times H \rightarrow H$  specify the transformation properties of matter  $\phi^A$  and tetrad  $e^a_\mu$  with respect to Lorentz and internal symmetries:

- Choose the group  $G = SO(3,1) \times SU(3) \times SU(2) \times U(1)$ . Then, for example, given any  $g \in G$  and a doublet

$$\begin{pmatrix} u_b \\ d_b \end{pmatrix}_L,$$

the action  $g \triangleright u_b$  encodes that  $u_b$  consists of 4 real-valued fields which transform as:

- a left-handed spinor wrt.  $SO(3,1)$ ,
  - as a “blue” component of the fundamental representation of  $SU(3)$ ,
  - and as “isospin  $+\frac{1}{2}$ ” of the left doublet wrt.  $SU(2) \times U(1)$ .
- Next choose the group  $H = \mathbb{R}^4$ . The action  $\triangleright$  of  $G$  on  $H$  is via vector representation for the  $SO(3,1)$  part and via trivial representation for the  $SU(3) \times SU(2) \times U(1)$  part.
  - Finally, the other maps in the 3-group are chosen to be trivial. For all  $l \in L$  and  $\vec{u}, \vec{v} \in H$ ,

$$\delta l = 1_H = 0, \quad \partial \vec{v} = 1_G, \quad \{\vec{u}, \vec{v}\} = 1_L.$$



# THE STANDARD MODEL

**The Standard Model 3-group,  $(L \xrightarrow{\delta} H \xrightarrow{\partial} G, \triangleright, \{-, -\})$ , defined as:**

$$G = SO(3, 1) \times SU(3) \times SU(2) \times U(1), \quad H = \mathbb{R}^4,$$

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}).$$

**The constrained 3BF action for the Standard Model coupled to Einstein-Cartan gravity:**

$$\begin{aligned} S_{GR+SM} = & \int_{\mathcal{M}_4} B_{\hat{\alpha}} \wedge \mathcal{F}^{\hat{\alpha}} + e_{\hat{a}} \wedge \mathcal{G}^{\hat{a}} + \phi_{\hat{A}} \wedge \mathcal{H}^{\hat{A}} \\ & + \left( B_{\hat{\alpha}} - C_{\hat{\alpha}}^{\hat{\beta}} M_{cd\hat{\beta}} e^c \wedge e^d \right) \wedge \lambda^{\hat{\alpha}} - \left( \gamma_{\hat{A}} - e^a \wedge e^b \wedge e^c C_{\hat{A}}^{\hat{B}} M_{abc\hat{B}} \right) \wedge \lambda^{\hat{A}} - 4\pi i l_p^2 \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c \phi_{\hat{A}} T^{d\hat{A}}_{\hat{B}} \phi^{\hat{B}} \\ & + \zeta^{ab}_{\hat{\alpha}} \wedge \left( M_{ab}^{\hat{\alpha}} \varepsilon^{cdef} e_c \wedge e_d \wedge e_e \wedge e_f - F^{\hat{\alpha}} \wedge e_c \wedge e_d \right) + \zeta^{ab}_{\hat{A}} \wedge \left( M_{abc}^{\hat{A}} \varepsilon^{cdef} e_d \wedge e_e \wedge e_f - F^{\hat{A}} \wedge e_a \wedge e_b \right) \\ & - \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d \left( \Lambda + M_{\hat{A}\hat{B}} \phi^{\hat{A}} \phi^{\hat{B}} + Y_{\hat{A}\hat{B}\hat{C}} \phi^{\hat{A}} \phi^{\hat{B}} \phi^{\hat{C}} + L_{\hat{A}\hat{B}\hat{C}\hat{D}} \phi^{\hat{A}} \phi^{\hat{B}} \phi^{\hat{C}} \phi^{\hat{D}} \right). \end{aligned}$$

$\Rightarrow$  Finally, one can go even further and separate scalar and fermion fields into distinct groups, employing the structure of a 4-group and a 4BF action. [Miković, MV (2021)]

# QUANTIZATION

*Revisit the spinfoam quantization method:*

- Step 1: Rewrite the GR+SM action... **done!**
- Step 2: Quantize the topological sector... **done!** (almost) [Radenković, MV (2022)]

$$\begin{aligned}
 Z = & |G|^{-|v|+|e|-|\Delta|} |H|^{|v|-|e|+|\Delta|-|\tau|} |L|^{-|v|+|e|-|\Delta|+|\tau|-|\sigma|} \\
 & \prod_{e \in T} \int_G dg_e \prod_{\Delta \in T} \int_H dh_\Delta \prod_{\tau \in T} \int_L dl_\tau \\
 & \prod_{\Delta \in T} \delta_G( \partial(h_\Delta) g_1 g_2 g_3^{-1} ) \prod_{\tau \in T} \delta_H( \delta(l_\tau) h_1 (g \triangleright h_2) h_3^{-1} h_4^{-1} ) \\
 & \prod_{\sigma \in T} \delta_L( l_1^{-1} h_1 \triangleright' \{h_2, (g_1 g_2) \triangleright h_3\} l_2^{-1} (h_4 \triangleright' l_3) l_4 h_5 \triangleright' (g_3 \triangleright l_5) )
 \end{aligned}$$

**Invariant wrt. 4D Pachner moves!**

- Step 3: Impose the simplicity constraints... **work in progress!**

# QUANTIZATION

**GR without matter can be described using 2-groups** ( $H \xrightarrow{\partial} G, \triangleright$ ):

- Topological  $2BF$  theory developed and studied: [Girelli, Pfeiffer, Popescu (2008)]  
[Miković, Martins (2011)]

$$S_{2BF} = \int_{\mathcal{M}_4} B^{ab} \wedge \mathcal{F}_{ab} + C^a \wedge \mathcal{G}_a .$$

- The choice  $G = SO(3, 1)$ ,  $H = \mathbb{R}^4$ , is called the *Poincaré 2-group*. The action for GR is

$$S_{GR} = \int_{\mathcal{M}_4} B^{ab} \wedge R_{ab}(\omega) + e^a \wedge G_a - \phi_{ab} \wedge \left( B^{ab} - \frac{1}{16\pi l_p^2} \varepsilon^{abcd} e_c \wedge e_d \right) .$$

One possible quantization prescription leads to the *spincube model*. [Miković, MV (2012)]

- Representation theory for 2-groups (including the Poincaré 2-group), has been developed in great detail. [Baez, Baratin, Freidel, Wise (2012)]
- The topological invariant and TQFT for the *Euclidean 2-group* ( $G = SO(4)$ ,  $H = \mathbb{R}^4$ ) has also been studied in detail. [Baratin, Freidel (2015)]  
[Asante, Dittrich, Girelli, Riello, Tsimiklis (2020)]

# CONCLUSIONS

- Higher gauge theory represents a formalism where gravity, gauge fields, fermions and Higgs are treated on an equal footing.
- Resulting generalized spinfoam models naturally include matter fields coupled to gravity.
- The underlying algebraic structure of a 3-group classifies all fundamental fields by specifying groups  $L, H, G$  and their maps  $\delta, \partial, \triangleright, \{-, -\}$ .
- This structure has natural geometrical interpretation of parallel transport along a curve, a surface, and a volume.
- The gauge group  $L$  specifies the complete matter sector of the Standard Model if one chooses

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) .$$

- The action  $\triangleright$  of  $G$  on  $L$  specifies the transformation properties of matter fields.
- Nontrivial choices of the 3-group structure may provide new avenues for research on unification of all fields.

***THANK YOU!***