Symmetry in quantum gravity via n-groups

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TOPICS

- **Introduction**
- **Higher category theory**
- **Lie 3-groups**
- **Higher gauge theory**
- **The Standard Model**
- **Quantization**
- **Conclusions**

INTRODUCTION

A short recap of the spinfoam quantization method:

- Step 1: Rewrite the GR action — as a topological BF theory plus simplicity constraint,

$$S_{\text{Plebanski}}[B,\omega,\phi] = \int_{\mathcal{M}_4} \langle B \wedge F(\omega)\rangle_{\mathfrak{g}} + \langle \phi(B\wedge B)\rangle_{\mathfrak{g}}\,,$$

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- Step 2: Quantize the topological sector — a state sum over a triangulated manifold $T(\mathcal{M}_4)$,

$$Z_{BF} = \sum_{\Lambda} \prod_{v} \mathcal{A}_v(\Lambda) \prod_{e} \mathcal{A}_e(\Lambda) \prod_{\Delta} \mathcal{A}_\Delta(\Lambda) \prod_{\tau} \mathcal{A}_\tau(\Lambda) \prod_{\sigma} \mathcal{A}_\sigma(\Lambda)\,.$$

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$$Z_{BF} \to Z_{GR} : \qquad \mathcal{A}(\Lambda) \to W(j) \,, \qquad j = f(\Lambda) \,,$$

  obtaining a state sum Z_{GR} which defines a spinfoam model (Barret-Crane, EPRL/FK, etc).

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Main problems:

- Tetrads are absent!! — no way to couple matter to gravity.
- Action for matter is not in the form “BF plus constraints” — no way to construct Z_{BF}.
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Solution — employ categorical ladder and higher gauge theory!

Use HGT to generalize:

- a *group* to an *n-group*,
- a *BF action* to an *nBF action*.

HIGHER CATEGORY THEORY

A flash introduction to higher category theory:

- An n-category is a set of *objects* with:
  - *morphisms* (maps between objects),
  - 2-*morphisms* (maps between morphisms),
  - 3-*morphisms* (maps between 2-morphisms), ... up to n-*morphisms*,

  along with certain axioms to provide suitable rules for composition, associativity, etc.

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The purpose of n-groups (for physicists):

⇒ *more fine-grained description of symmetry* using an n-group, than using a group,

⇒ generalization of differential geometry: *parallel transport, connection, holonomy, curvature.*

LIE 3-GROUPS

Focus on a Lie 3-group, specified in detail by a 2-crossed module:

[Faria Martins, Picken (2011); Wang (2014)]

$$\left(\quad L \xrightarrow{\delta} H \xrightarrow{\partial} G \quad , \quad \triangleright \quad , \quad \{\_\,,\_\} \quad \right)$$

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- L, H, G — Lie groups,
- δ, ∂ — boundary morphisms,
- $\triangleright$ — action of G, $\qquad \triangleright : G \times G \to G, \quad \triangleright : G \times H \to H, \quad \triangleright : G \times L \to L,$
- $\{\_\,,\_\}$ — Peiffer lifting, $\qquad \{\_\,,\_\} : H \times H \to L.$

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Axioms that hold among these maps:

Chain complex:	$\partial\delta = 1_G$,
Conjugation:	$g \triangleright g_0 = g\, g_0\, g^{-1}$,
G-equivariance of ∂ and δ:	$g \triangleright \partial h = \partial(g \triangleright h)\,, \quad g \triangleright \delta l = \delta(g \triangleright l)$,
G-equivariance of lifting:	$g \triangleright \{h_1, h_2\} = \{g \triangleright h_1, g \triangleright h_2\}$,
Peiffer commutator:	$\delta\,\{h_1, h_2\} = h_1 h_2 h_1^{-1} (\partial h_1) \triangleright h_2^{-1}$,
L-commutator:	$\{\delta l_1, \delta l_2\} = l_1 l_2 l_1^{-1} l_2^{-1}$,
δ-lifting relation:	$\{\delta l, h\}\,\{h, \delta l\} = l(\partial h \triangleright l^{-1})$,
Left product rule:	$\{h_1 h_2, h_3\} = \{h_1, h_2 h_3 h_2^{-1}\}\, \partial h_1 \triangleright \{h_2, h_3\}$.

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Purpose of all this — to generalize the notion of parallel transport, from curves to surfaces to volumes:

- Connection generalized to a 3-connection (α, β, γ), a triple of algebra-valued differential forms:

$$
\begin{array}{rcrll}
\alpha & = & \alpha^{\alpha}{}_{\mu}(x) & \tau_{\alpha} \otimes \mathbf{d}x^{\mu} & \in \mathfrak{g} \otimes \Lambda^{1}(\mathcal{M})\,, \\
\beta & = & \frac{1}{2}\,\beta^{a}{}_{\mu\nu}(x) & t_{a} \otimes \mathbf{d}x^{\mu} \wedge \mathbf{d}x^{\nu} & \in \mathfrak{h} \otimes \Lambda^{2}(\mathcal{M})\,, \\
\gamma & = & \frac{1}{3!}\,\gamma^{A}{}_{\mu\nu\rho}(x) & T_{A} \otimes \mathbf{d}x^{\mu} \wedge \mathbf{d}x^{\nu} \wedge \mathbf{d}x^{\rho} & \in \mathfrak{l} \otimes \Lambda^{3}(\mathcal{M})\,.
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- Line holonomy generalized to surface and volume holonomies:

$$g = \mathcal{P}\exp \int_{\mathcal{P}_1} \alpha\,, \qquad h = \mathcal{S}\exp \int_{\mathcal{S}_2} \beta\,, \qquad l = \mathcal{V}\exp \int_{\mathcal{V}_3} \gamma\,.$$

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- Ordinary curvature generalized to 3-curvature $(\mathcal{F}, \mathcal{G}, \mathcal{H})$, where:

$$
\begin{array}{rcl}
\mathcal{F} & = & \mathbf{d}\alpha + \alpha \wedge \alpha - \partial\beta\,, \\
\mathcal{G} & = & \mathbf{d}\beta + \alpha \wedge^{\triangleright} \beta - \delta\gamma\,, \\
\mathcal{H} & = & \mathbf{d}\gamma + \alpha \wedge^{\triangleright} \gamma - \{\beta \wedge \beta\}\,.
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HIGHER GAUGE THEORY

At this point one can construct the action for a higher gauge theory:

$$S_{3BF} = \int_{\mathcal{M}_4} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} + \langle C \wedge \mathcal{G} \rangle_{\mathfrak{h}} + \langle D \wedge \mathcal{H} \rangle_{\mathfrak{l}} \,.$$

$\Rightarrow$ Topological $3BF$ theory, based on the 3-group $(\, L \xrightarrow{\delta} H \xrightarrow{\partial} G \,,\, \triangleright \,,\, \{\_\,,\_\} \,)$.

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The physical interpretation of the Lagrange multipliers C and D:

- for $H = \mathbb{R}^4$, multiplier C can be ***interpreted as the tetrad*** 1***-form***:

$$C \quad \to \quad e \quad = \quad e^a{}_\mu(x)\, t_a \otimes \mathbf{d}x^\mu \,,$$

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$\Rightarrow$ The action thus becomes:

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The fermion sector gives us:

$$\left.\begin{matrix} \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L & \begin{pmatrix}u_r \\ d_r\end{pmatrix}_L & \begin{pmatrix}u_g \\ d_g\end{pmatrix}_L & \begin{pmatrix}u_b \\ d_b\end{pmatrix}_L \\ \\ (\nu_e)_R & (u_r)_R & (u_g)_R & (u_b)_R \\ \\ (e^-)_R & (d_r)_R & (d_g)_R & (d_b)_R \end{matrix}\right\} = 16 \ \frac{\text{spinors}}{\text{family}} \times$$

$$\times 3 \text{ families } \ \times 4 \ \frac{\text{real-valued components}}{\text{spinor}} \ = 192 \text{ real-valued components } \phi^A \, .$$

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This suggests the structure for L in the form:

$$L = L_{\text{fermion}} \times L_{\text{Higgs}} \,, \qquad \dim L_{\text{fermion}} = 192 \,, \qquad \dim L_{\text{Higgs}} = 4 \,.$$

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- Choose the group $G = SO(3,1) \times SU(3) \times SU(2) \times U(1)$. Then, for example, given any $g \in G$ and a doublet

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  the action $g \triangleright u_b$ encodes that u_b consists of 4 real-valued fields which transform as:

  - a left-handed spinor wrt. $SO(3,1)$,
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$$L_{\text{fermion}} = L_{\text{1st family}} \times L_{\text{2nd family}} \times L_{\text{3rd family}} , \qquad \dim L_{k\text{-th family}} = 64 .$$

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- Next choose the group $H = \mathbb{R}^4$. The action $\triangleright$ of G on H is via vector representation for the $SO(3,1)$ part and via trivial representation for the $SU(3) \times SU(2) \times U(1)$ part.

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The other maps in the 3-group are chosen to be trivial:

- For all $l \in L$ and $\vec{u}, \vec{v} \in H$, we define

$$\delta l = 1_H = 0\,, \qquad \partial\vec{v} = 1_G\,, \qquad \{\vec{u}, \vec{v}\} = 1_L\,.$$

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- In order to satisfy all axioms of a 3-group, the group L must be Abelian (the L-commutator axiom). Thus, given the Abelian nature and dimensionality of L, the simplest choices for its component groups are

$$L_{\text{Higgs}} = \mathbb{R}^4(\mathbb{C}) \,, \qquad L_{\text{fermion family}} = \mathbb{R}^{64}(\mathbb{G}) \,,$$

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The Standard Model 3-group is thus specified with the choice:

$$G = SO(3,1) \times SU(3) \times SU(2) \times U(1)\,, \qquad H = \mathbb{R}^4\,,$$

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G})\,,$$

and with the action $\triangleright$ of G on H, L as previously described.

THE STANDARD MODEL

The Standard Model 3***-group,*** $\left(L \overset{\delta}{\to} H \overset{\partial}{\to} G \,,\, \triangleright \,,\, \{\_\,,\,\_\} \right)$***, defined as:***

$$G = SO(3,1) \times SU(3) \times SU(2) \times U(1)\,, \qquad H = \mathbb{R}^4\,,$$

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G})\,.$$

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The constrained $3BF$ ***action for the Standard Model coupled to Einstein-Cartan gravity:***

$$S_{GR+SM} = \int_{\mathcal{M}_4} B_{\hat{\alpha}} \wedge \mathcal{F}^{\hat{\alpha}} + e_{\hat{a}} \wedge \mathcal{G}^{\hat{a}} + \phi_{\hat{A}} \wedge \mathcal{H}^{\hat{A}}$$

$$+ \left(B_{\hat{\alpha}} - C_{\hat{\alpha}}{}^{\hat{\beta}} M_{cd\hat{\beta}} e^c \wedge e^d\right) \wedge \lambda^{\hat{\alpha}} - \left(\gamma_{\hat{A}} - e^a \wedge e^b \wedge e^c C_{\hat{A}}{}^{\hat{B}} M_{abc\hat{B}}\right) \wedge \lambda^{\hat{A}} - 4\pi i\, l_p^2\, \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c \phi_{\hat{A}} T^{d\hat{A}}{}_{\hat{B}} \phi^{\hat{B}}$$

$$+ \zeta^{ab}{}_{\hat{\alpha}} \wedge \left(M_{ab}{}^{\hat{\alpha}} \varepsilon^{cdef} e_c \wedge e_d \wedge e_e \wedge e_f - F^{\hat{\alpha}} \wedge e_c \wedge e_d\right) + \zeta^{ab}{}_{\hat{A}} \wedge \left(M_{abc}{}^{\hat{A}} \varepsilon^{cdef} e_d \wedge e_e \wedge e_f - F^{\hat{A}} \wedge e_a \wedge e_b\right)$$

$$- \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d \,\left(\Lambda + M_{\hat{A}\hat{B}} \phi^{\hat{A}} \phi^{\hat{B}} + Y_{\hat{A}\hat{B}\hat{C}} \phi^{\hat{A}} \phi^{\hat{B}} \phi^{\hat{C}} + L_{\hat{A}\hat{B}\hat{C}\hat{D}} \phi^{\hat{A}} \phi^{\hat{B}} \phi^{\hat{C}} \phi^{\hat{D}}\right)\,.$$

$\Rightarrow$ Finally, one can go even further and separate scalar and fermion fields into distinct groups, employing the structure of a 4-group and a $4BF$ action. [Miković, MV (2021)]

QUANTIZATION

Revisit the spinfoam quantization method:

- Step 1: Rewrite the GR+SM action… **done!**
- Step 2: Quantize the topological sector… **done!** (see Tijana's talk)
- Step 3: Impose the simplicity constraints… **work in progress!**

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GR without matter can be described using 2-groups $(H \stackrel{\partial}{\to} G, \triangleright)$***:***

- The choice $G = SO(3,1)$, $H = \mathbb{R}^4$, is called the *Poincaré 2-group*. The corresponding constrained $2BF$ action for GR is

$$S_{GR} = \int_{\mathcal{M}_4} B^{ab} \wedge R_{ab}(\omega) + e^a \wedge G_a - \phi_{ab} \wedge \left(B^{ab} - \frac{1}{16\pi l_p^2} \varepsilon^{abcd} \, e_c \wedge e_d \right)$$

  One possible quantization prescription leads to the *spincube model.* [Miković, MV (2012)]

- A detailed representation theory for 2-groups (including the Poincaré 2-group), has been developed in great detail. [Baez, Baratin, Freidel, Wise (2012)]
- The topological invariant and TQFT for the *Euclidean 2-group* ($G = SO(4)$, $H = \mathbb{R}^4$) has also been studied in detail. [Baratin, Freidel (2015); Asante etal (2020)]

CONCLUSIONS

- Higher gauge theory represents a formalism where gravity, gauge fields, fermions and Higgs are treated on an equal footing.
- Resulting generalized spinfoam models naturally include matter fields coupled to gravity.
- The underlying algebraic structure of a 3-group classifies all fundamental fields by specifying groups L, H, G and their maps $\delta, \partial, \triangleright, \{\_\,,\_\}$.
- This structure has natural geometrical interpretation of parallel transport along a curve, a surface, and a volume.
- The gauge group L specifies the complete matter sector of the Standard Model if one chooses

$$L = \mathbb{R}^4(\mathbb{C}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G}) \times \mathbb{R}^{64}(\mathbb{G})\,.$$

- The action $\triangleright$ of G on L specifies the transformation properties of matter fields.
- Nontrivial choices of the 3-group structure may provide new avenues for research on unification of all fields.

THANK YOU!