A REVIEW OF SOME RESEARCH PROGRAMS IN CLASSICAL AND QUANTUM GRAVITY

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TOPICS

- **Influence of curvature and torsion on the motion of bodies**
- **Properties of spinfoam models of Loop Quantum Gravity**
- **Constructions of QG models based on higher gauge theories**
- **Quantum information theoretical approach to quantum gravity**

MOTION OF BODIES

Collaboration with Prof. M. Vasilić, 7 papers, 2004–2010:

[1] Classical string in curved backgrounds
M. Vasilić and M. Vojinović, *Phys. Rev. D* **73**, 124013 (2006).

[2] Classical spinning branes in curved backgrounds
M. Vasilić and M. Vojinović, *JHEP* **07**, 028 (2007).

[3] Single-pole interaction of the particle with the string
M. Vasilić and M. Vojinović, *SIGMA* **4**, 019 (2008).

[4] Interaction of particle with the string in pole-dipole approximation
M. Vasilić and M. Vojinović, *Fortschr. Phys.* **56**, 542 (2008).

[5] Zero-size objects in Riemann-Cartan spacetime
M. Vasilić and M. Vojinović, *JHEP* **08**, 104 (2008).

[6] Spinning branes in Riemann-Cartan spacetime
M. Vasilić and M. Vojinović, *Phys. Rev. D* **78**, 104002 (2008).

[7] Test membranes in Riemann-Cartan spacetimes
M. Vasilić and M. Vojinović, *Phys. Rev. D* **81**, 024025 (2010).

MOTION OF BODIES

Main problem: what are the effective classical equations of motion for a "body", made of some matter, moving freely in spacetime with nonzero curvature and torsion?

- Describe a "body" as a spatially localized configuration of matter fields, a *kink*.
- Impose local Poincaré invariance of the matter fields that make up the kink.
- In spacetime with curvature and torsion, second Noether theorem gives rise to covariant conservation of stress-energy and spin tensors.
- Impose covariant conservation laws to deduce:
  - equations of motion for the kink,
  - structure equations for the stress-energy and spin tensors.

Key insight: in the context of general relativity, the equations of motion for the kink are derived as a consequence of Einstein equations (and similarly in Einstein-Cartan gravity).

MOTION OF BODIES

The new math result, underpinning the work — Dirac δ series:

- Given a function $f(x)$ which is "well localized" around a peak x_0, one can expand it into a series of derivatives of Dirac δ function, around the point z, as

$$f(x) = b_0\delta(x-z) + b_1\delta'(x-z) + b_2\delta''(x-z) + \cdots = \sum_{n=0}^{\infty} b_n \frac{d^n}{dx^n}\delta(x-z)\,.$$

- The coefficients b_n are given by the inverse transformation, and are known as n-*th moments* of f:

$$b_n = \frac{(-1)^n}{n!}\int_{\mathbb{R}} dx\,(x-z)^n f(x)\,.$$

- Crucial property: iff $z \approx x_0$, we have $b_0 \gg b_1 \gg b_2 \gg b_3 \gg \ldots$
- The δ series can be generalized to describe a p-dimensional kink in D-dimensional ambient spacetime:

$$f(x) = \int_{\Sigma} d^{p+1}\xi\sqrt{-\gamma}\,\sum_{n=0}^{\infty} \nabla_{\mu_1}\ldots\nabla_{\mu_n}\left[B^{\mu_1\ldots\mu_n}(\xi)\frac{\delta^{(D)}(x-z(\xi))}{\sqrt{-g}}\right]\,.$$

MOTION OF BODIES

The δ series can be applied to describe the stress-energy and spin tensors of the p-dimensional kink:

$$\tau^{\mu\nu}(x) = \int_{\Sigma} d^{p+1}\xi\sqrt{-\gamma}\,\sum_{n=0}^{\infty}\nabla_{\rho_1}\ldots\nabla_{\rho_n}\left[B^{\mu\nu\rho_1\ldots\rho_n}(\xi)\frac{\delta^{(D)}(x-z(\xi))}{\sqrt{-g}}\right],$$

$$\sigma^{\lambda\mu\nu}(x) = \int_{\Sigma} d^{p+1}\xi\sqrt{-\gamma}\,\sum_{n=0}^{\infty}\nabla_{\rho_1}\ldots\nabla_{\rho_n}\left[C^{\lambda\mu\nu\rho_1\ldots\rho_n}(\xi)\frac{\delta^{(D)}(x-z(\xi))}{\sqrt{-g}}\right].$$

- To derive equations of motion, one cuts the series after a finite number of terms in the series (*multipole approximation*), which places a restriction on the expansion hypersurface Σ to coincide with the "localization peak" of matter fields.
- Then enforce the covariant conservation equations from second Noether theorem:

$$(\nabla_\nu + \mathcal{T}^{\lambda}{}_{\nu\lambda})\tau^{\nu}{}_{\mu} = \tau^{\nu}{}_{\rho}\mathcal{T}^{\rho}{}_{\mu\nu} + \frac{1}{2}\sigma^{\nu\rho\sigma}\mathcal{R}_{\rho\sigma\mu\nu}\,,$$

$$(\nabla_\nu + \mathcal{T}^{\lambda}{}_{\nu\lambda})\sigma^{\nu}{}_{\rho\sigma} = \tau_{\rho\sigma} - \tau_{\sigma\rho}\,.$$

MOTION OF BODIES

The result:

- Restriction on the expansion hypersurface Σ, coupled with covariant conservation equations, transforms into a differential equation for $z(\xi)$, i.e., an *equation of motion of the kink*. In the pole-dipole approximation:

$$\nabla_b\Big[m^{ab}u^\mu_a - 2u^b_\lambda(\nabla_a J^{\mu\lambda a} + D^{\mu\lambda}) + u^\mu_c u^c_\rho u^b_\lambda(\nabla_a J^{\rho\lambda a} + D^{\rho\lambda})\Big] =$$

$$= u^\nu_a J^{\lambda\rho a} R^\mu{}_{\nu\lambda\rho} + \frac{1}{2} C_{\nu\rho\lambda}\nabla^\mu K^{\rho\lambda\nu}\,.$$

- Additionally, we obtain an equation of motion for the angular momentum field of the kink,

$$P_\perp{}^\mu_\lambda P_\perp{}^\nu_\rho(\nabla_a J^{\lambda\rho a} + D^{\lambda\rho}) = 0\,,$$

- the structure equations for the multipole moments in terms of the free parameters,

$$B^{\mu\nu} = m^{ab}u^\mu_a u^\nu_b + \nabla_a N^{\mu\nu a} + 2u^a_\lambda u^{(\mu}_a P_\perp{}^{\nu)}_\rho \nabla_a J^{\lambda\rho a}\,, \quad B^{\mu\nu\rho} = 2u^{(\mu}_a J^{\nu)\rho a} + N^{\mu\nu a}u^\rho_a\,,$$

- and appropriate boundary conditions at the boundary hypersurface $\partial\Sigma$, if it exists.

MOTION OF BODIES

Further results and applications:

- Special cases of equations of motion — geodesic equation for a test particle, Nambu-Goto equation for a string, Papapetrou equations for a particle with spin, and many more general situations. . .
- Various gauge symmetries of the equations of motion have been studied — spacetime diffeomorphisms, diffeomorphisms of worldsheet hypersurface Σ, and also two additional symmetries corresponding to the level of multipole approximation, and to the choice of the central surface of mass of the kink.
- Applications of the formalism include the study of the Kalb-Ramond field in the string theory effective action, etc.
- The formalism has also been applied in quantum gravity, to study the violation of the weak equivalence principle for a particle in superposed gravitational fields.
- The formalism has been extended (by other researchers) to include electric charge density and the motion in an external electromagnetic field, and further to the Yang-Mills case, etc. . .

PROPERTIES OF SPINFOAM MODELS

Collaboration with Prof. A. Miković, 6 papers, 2009–2016:

[8] Large-spin asymptotics of Euclidean LQG flat-space wavefunctions
A. Miković and M. Vojinović, *Adv. Theor. Math. Phys.* **15**, 801 (2011).

[9] Effective action and semiclassical limit of spin foam models
A. Miković and M. Vojinović, *Class. Quant. Grav.* **28**, 225004 (2011).

[10] A finiteness bound for the EPRL/FK spin foam model
A. Miković and M. Vojinović, *Class. Quant. Grav.* **30**, 035001 (2013).

[11] Cosine problem in EPRL/FK spin foam model
M. Vojinović, *Gen. Relativ. Gravit.* **46**, 1616 (2014).

[12] Solution to the cosmological constant problem in a Regge quantum gravity model
A. Miković and M. Vojinović, *Europhys. Lett.* **110**, 40008 (2015).

[13] Causal dynamical triangulations in the spincube model of quantum gravity
M. Vojinović, *Phys. Rev. D* **94**, 024058 (2016).

PROPERTIES OF SPINFOAM MODELS

Main problems:

- How to ensure the UV and IR finiteness of a spinfoam model?
- Does a spinfoam model have a correct semiclassical limit?
- How to compute IR observables in quantum gravity? Are there any interesting ones?

Additional problems:

- What is the relationship between spinfoam models and other comparable approaches to quantum gravity, such as causal dynamical triangulations and causal set theory?

Key insight: all previously constructed spinfoam models had some unneccesary restrictions imposed on them. Relaxing those restrictions leads to more general models, which feature simple solutions to above problems.

PROPERTIES OF SPINFOAM MODELS

The new math result, underpinning the work — nonperturbative effective action equation:

- In ordinary QFT, within the path integral formalism, one can prove the following nonperturbative result:

$$\exp\left(i\Gamma[\phi]\right) = \int \mathcal{D}\varphi \, \exp\left[iS[\phi+\varphi] - i\int d^4x\, \varphi(x)\, \frac{\delta\Gamma[\phi]}{\delta\phi(x)} \right] \,,$$

  where $S[\phi]$ is the classical action, $\Gamma[\phi]$ is the effective action (with all quantum corrections taken into account).

- The idea of an effective action is based on the *background field method*. Here ϕ is the background field, in arbitrary configuration. The effective action equation is nontrivial for off-shell background fields, since then $\frac{\delta\Gamma[\phi]}{\delta\phi(x)} \neq 0$.

The effective action equation is straightforward to generalize from smooth to piecewise-linear configurations, and further to spinfoam models, and beyond to other models of quantum gravity.

PROPERTIES OF SPINFOAM MODELS

Generic state sum model represents a Feynman-discretized definition of the path integral. In 4 dimensions, we have:

$$Z \equiv \int \mathcal{D}\varphi \, e^{iS[\varphi]} = \sum_{\varphi} \prod_{v} \mathcal{A}_v(\varphi) \prod_{\epsilon} \mathcal{A}_\epsilon(\varphi) \prod_{\triangle} \mathcal{A}_\triangle(\varphi) \prod_{\tau} \mathcal{A}_\tau(\varphi) \prod_{\sigma} \mathcal{A}_\sigma(\varphi) \,.$$

- The fields ϕ are labels associated to the elements of a triangulation of 4D spacetime — vertices v, edges ϵ, triangles $\triangle$, tetrahedra τ and 4-simplices σ. In general, each subsimplex in the triangulation has its amplitude function $\mathcal{A}(\phi)$, which can be chosen *freely*.

- Given a state sum Z, one can define the effective action equation by substituting:

$$Z \to \exp(i\Gamma[\phi]) \,, \qquad \mathcal{A}_*(\varphi) \to \mathcal{A}_*(\phi + \varphi) \,, \qquad * \in \{v, \epsilon, \triangle, \tau\} \,,$$

$$\mathcal{A}_\sigma(\varphi) \to \mathcal{A}_\sigma(\phi + \varphi) \exp\left(-i\varphi_\sigma \frac{\delta\Gamma[\phi]}{\delta\phi_\sigma} \right) \,.$$

One can employ the effective action equation to all spinfoam models and efficiently study their properties and all of the above problems.

PROPERTIES OF SPINFOAM MODELS

Results:

- A spinfoam model can have a correct semiclassical limit by suitable choice of the amplitudes $\mathcal{A}(\phi)$ and evaluating the effective action in the large-distance limit.
- The state sum Z can be IR finite by a suitable choice of the "measure terms" (amplitudes $\mathcal{A}(\phi)$ for subleading simplices). Also, it is UV finite by construction.
- In the evaluation of the effective action, one can extract terms of the form

$$\prod_\sigma \exp\left(i\Lambda\, {}^{(4)}V_\sigma(l)\right) = \exp\left(i\Lambda \sum_\sigma {}^{(4)}V_\sigma(l)\right) \approx \exp\left(i\Lambda \int d^4x\sqrt{-g}\right) ,$$

giving rise to the evaluation of the cosmological constant term:

$$\Gamma[l,\phi] \propto \int d^4x\sqrt{-g}\,\Lambda\,.$$

It turns out that this procedure can give a nonperturbative result $0 < \Lambda l_p^2 \ll 1$, which fits well with the experimental result $\Lambda l_p^2 \approx 10^{-122}$.

HIGHER GAUGE THEORIES

Collaboration with Prof. A. Miković and students M. A. Oliveira, T. Radenković, P. Stipsić and M. Đorđević, 6 papers, 2011–now:

[14] Poincaré 2-group and quantum gravity
A. Miković and M. Vojinović, *Class. Quant. Grav.* **29**, 165003 (2012).

[15] Hamiltonian analysis of the BFCG theory for the Poincaré 2-group
A. Miković, M. A. Oliveira and M. Vojinović, *Class. Quant. Grav.* **33**, 065007 (2016).

[16] Hamiltonian analysis of the BFCG formulation of general relativity
A. Miković, M. A. Oliveira and M. Vojinović, *Class. Quant. Grav.* **36**, 015005 (2019).

[17] Higher gauge theories based on 3-groups
T. Radenković and M. Vojinović, *JHEP* **10**, 222 (2019).

[18] Hamiltonian Analysis for the Scalar Electrodynamics as 3BF Theory
T. Radenković and M. Vojinović, *Symmetry* **12**, 620 (2020).

[19] Standard Model and 4-groups
A. Miković and M. Vojinović, *Europhys. Lett.* **133**, 61001 (2021).

HIGHER GAUGE THEORIES

Main problem: how to couple matter fields to gravity in spinfoam models?

- The starting point for the construction of all spinfoam models is the BF theory, based on the Lorentz group, $SO(3,1)$.
- As a consequence, gravity is described by the spin-connection $\omega^{ab}{}_{\mu}(x)$, while tetrads $e^a{}_{\mu}(x)$ appear exclusively on-shell.
- Thus, since some matter fields couple directly to tetrads, it is impossible to include them off-shell (as new fields in the path integral).

Key insight: higher gauge theory generalizes BF action to the $BFCG$ action, where the C term behaves precisely like the tetrad, $C \equiv e$! Therefore, $2BF$ theory contains tetrads off-shell, and in this way facilitates the coupling of matter fields.

HIGHER GAUGE THEORIES

The new math result, underpinning the work — categorical ladder and n-groups:

- An n-group ($n = 1, 2, 3, 4$) is a category with a single object, and all k-morphisms invertible.
- n-groups are equivalent to $(n-1)$-crossed modules, whose differential versions play the role of a "Lie algebra" structure. For example, a 2-group is isomorphic to a crossed module, $(H \stackrel{\partial}{\to} G\,,\, \triangleright)$, while a 3-group is isomorphic to a 2-crossed module, $(L \stackrel{\delta}{\to} H \stackrel{\partial}{\to} G\,,\, \triangleright\,,\, \{\_\,,\_\})$.
- n-groups give rise to generalized parallel transport, holonomy, and connection:

$$
\begin{array}{lcll}
\alpha & = & \alpha^{\alpha}{}_{\mu}(x)\, \tau_{\alpha}\, \mathbf{d}x^{\mu} & \in \Lambda^1(\mathcal{M}, \mathfrak{g})\,, \\
\beta & = & \frac{1}{2}\beta^{a}{}_{\mu\nu}(x)\, t_a\, \mathbf{d}x^{\mu} \wedge \mathbf{d}x^{\nu} & \in \Lambda^2(\mathcal{M}, \mathfrak{h})\,, \\
\gamma & = & \frac{1}{3!}\gamma^{A}{}_{\mu\nu\rho}(x)\, T_A\, \mathbf{d}x^{\mu} \wedge \mathbf{d}x^{\nu} \wedge \mathbf{d}x^{\rho} & \in \Lambda^3(\mathcal{M}, \mathfrak{l})\,.
\end{array}
$$

n-groups can be used to describe gauge symmetry, giving rise to topological nBF actions, used in construction of QG models.

HIGHER GAUGE THEORIES

In the 3**-group case,** $(L \stackrel{\delta}{\to} H \stackrel{\partial}{\to} G\,, \triangleright\,, \{\_\,,\_\})$**, the corresponding** $3BF$ **action is:**

$$S_{3BF} = \int_{\mathcal{M}} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} + \langle C \wedge \mathcal{G} \rangle_{\mathfrak{h}} + \langle D \wedge \mathcal{H} \rangle_{\mathfrak{l}}\,.$$

- $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$ are curvatures for the 3-connection forms α, β, γ, while B, C, D are Lagrange multipliers.
- Gauge fields are described by the connection 1-form α. A typical choice of the group G could be $G = SO(3,1) \times SU(3) \times SU(2) \times U(1)$.
- The tetrad fields are described by the Lagrange multiplier 1-form $C \to e$. This is typically achieved by choosing the group H to be spacetime translations, $H = \mathbb{R}^4$.
- The scalar and fermion fields are described by the Lagrange multiplier 0-form $D \to \phi, \psi, \bar{\psi}$. They are classified by a suitable choice of the group L.

The 3**-group and** 4**-group structures provide an algebraic classification of all fields in nature: gauge, gravitational, and matter fields — in constrast to the Standard Model!**

HIGHER GAUGE THEORIES

Results and future work:

- The constrained $3BF$ and $4BF$ actions have been constructed, describing the complete Standard Model of elementary particles, coupled to Einstein-Cartan gravity.
- Hamiltonian analysis has been performed for $2BF$ and $3BF$ actions, and all gauge symmetries have been studied.
- A topological invariant corresponding to $3BF$ action is under construction.
- Study of symmetry breaking mechanisms and higher gauge Noether theorems is in progress.
- Study of the n-group analogue of the Coleman-Mandula theorem is in progress.
- Construction of the full state sum model for quantum gravity with matter is in progress.

A promising research direction for grand unification and a "theory of everything".

QUANTUM INFORMATION QG

Collaboration with Dr. N. Paunković and students F. Pipa, R. Faleiro, V. Manojlović and J. Janjić, 3 papers, 2015–now:

[20] Gauge protected entanglement between gravity and matter
N. Paunković and M. Vojinović, *Class. Quant. Grav.* **35**, 185015 (2018).

[21] Entanglement-induced deviation from the geodesic motion in quantum gravity
F. Pipa, N. Paunković and M. Vojinović, *Jour. Cosmol. Astropart. Phys.* **09**, 057 (2019).

[22] Causal orders, quantum circuits and spacetime: distinguishing between definite and superposed causal orders
N. Paunković and M. Vojinović, *Quantum* **4**, 275 (2020).

QUANTUM INFORMATION QG

Main problem: what are the nontrivial applications of superposition principle and measurement postulate to gravity, and what are their consequences?

- Study curious conceptual situations — putting a gravitational field in the Schrödinger cat state, shooting a black hole through a beam splitter, detecting superpositions of different causal orders of events, etc. . .
- Also some foundational questions — operational description of spacetime emergence, entanglement with vacuum, quantum violation of weak equivalence principle. . .
- There is a sizable community of experts on quantum information theory, who are attacking the problem of quantum gravity using the techniques of quantum information and similar.

Key insight: the quantum information people lack expertise in field theory — an excellent situation for opportunistic joint research! :-)

QUANTUM INFORMATION QG

The new/old math result, underpinning the work — QM is 1-dimensional QFT:

- QFT is typically $(3+1)$-dimensional, formulated in the Heisenberg (or interaction) picture. When QM is rephrased in Heisenberg picture, it becomes equivalent to a $(0+1)$-dimensional QFT.
- This correspondence can be exploited to "upgrade" various typical QM notions to QFT and further to QG.
- It can also be exploited to "downgrade" various typical QFT notions to QM.
- A $(0+1)$-dimensional spacetime has no curvature (nor torsion), and therefore no gravity. It also features no spatial boundary conditions. A $(3+1)$-dimensional spacetime features both gravity and nontrivial boundary.

QUANTUM INFORMATION QG

Results and future work:

- Gravity and matter are coupled so that diffeomorphism-invariant states are never product states between gravity and matter, but essentially always entangled.
- A test particle travelling through spacetime which features a superposition of two different gravitational fields will experience a force pushing it off a geodesic trajectory of either gravitational field, thus violating the weak equivalence principle.
- Superposition of different orders of operations can be distinguished from the superposition of different causal orders in spacetime, with an explicit observable.
- Closed timelike curves can be constructed using a superposition of two globally hyperbolic spacetimes, with no need for exotic matter (in preparation).
- There is an operational protocol which can detect the existence of a spacetime manifold, and measure its dimension and topology. This (almost) establishes spacetime as an ontological reality (in preparation).

THANK YOU!