Symmetry protected entanglement between gravity and matter

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Abstract

We show that gravity and matter fields are always entangled, as a consequence of the local Poincaré symmetry. First, we present a generic argument, applicable to any particular theory of quantum gravity with matter, by performing the analysis in the abstract nonperturbative canonical framework, demonstrating that the scalar constraint allows for only the entangled states as the physical ones. Also, within the covariant framework, we show explicitly that the Hartle-Hawking state in the Regge model of quantum gravity is necessarily entangled. Our result is potentially relevant for the quantum-to-classical transition, taken within the framework of the decoherence programme: due to the symmetry requirements, the matter does not decohere, it is by default decohered by gravity. Generically, entanglement is a consequence of interaction. This new entanglement could potentially, in form of an "effective interaction", bring about corrections to the weak equivalence principle, further confirming that spacetime as a smooth four-dimensional manifold is an emergent phenomenon. Finally, the existence of the symmetry-protected entanglement between gravity and matter could be seen as a criterion for a plausible theory of quantum gravity, and in the case of perturbative quantisation approaches, a confirmation of the persistence of the manifestly broken symmetry.

Canonical quantization

The Hamiltonian of a gravity-matter quantum system [1]:

$$\hat{H} = \int_{\Sigma_3} d^3\vec{x} \Big[N\hat{\mathcal{C}} + N^i\hat{\mathcal{C}}_i + N^{ab}\hat{\mathcal{C}}_{ab} \Big]$$

The equations of motion for an observable $\hat{\mathcal{O}}$:

$$i\frac{\partial \hat{\mathcal{O}}(x)}{\partial t} = [\hat{\mathcal{O}}(x), \hat{H}]$$

Heisenberg picture!

Dirac's quantisation programme of constrained systems [2].

Local Poincaré gauge invariance conditions:

$$\hat{\mathcal{C}}|\Psi\rangle = 0 \qquad \hat{\mathcal{C}}_i|\Psi\rangle = 0 \qquad \hat{\mathcal{C}}_{ab}|\Psi\rangle = 0$$

Gupta-Bleuler-like conditions [3, 4].

The physical gauge-invariant Hilbert space is a proper subset of the total Hilbert space:

$$\mathcal{H}_{\text{phys}} \subset \mathcal{H}_G \otimes \mathcal{H}_M$$

Entanglement

Case by case examination of Lagrangians for the scalar, Dirac and Yang-Mills fields, minimally coupled to gravity.

The scalar constraint has the general form

$$\hat{\mathcal{C}} = \mathcal{C}_G(\hat{g}, \hat{\pi}_g) + \hat{\pi}_\phi \hat{\nabla}_\perp \hat{\phi} - \frac{1}{N}\mathcal{L}_M(\hat{g}, \hat{\pi}_g, \hat{\phi}, \hat{\pi}_\phi)$$

and the matter Lagrangian $\mathcal{L}_M$ mixes gravitational and matter fields in a complicated way, so the scalar constraint equation $\hat{\mathcal{C}}|\Psi\rangle = 0$ has no separable solutions:

$$|\Psi_G\rangle \otimes |\Psi_M\rangle \notin \mathcal{H}_{\text{phys}}$$

Covariant quantization

The path integral of a gravity-matter quantum system:

$$Z = \int \mathcal{D}g \int \mathcal{D}\phi \; e^{iS[g,\phi]}$$

Feynman's quantisation programme, applied to gravity and matter.

A generic state vector:

$$|\Psi\rangle = \int \mathcal{D}g \int \mathcal{D}\phi \; \Psi[g,\phi] \; |g\rangle \otimes |\phi\rangle$$

The density matrix of a partial matter state:

$$\hat{\rho}_M = \text{Tr}_G \, |\Psi\rangle \otimes \langle\Psi|$$

$\int \mathcal{D}g \, \langle g| \big(|\Psi\rangle \otimes \langle\Psi|\big)|g\rangle$

Entanglement

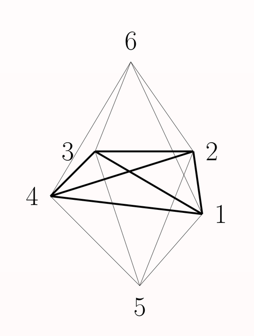
Analysis of an explicit example.

Define the path integral via the Regge quantum gravity toy example, with a single scalar field as matter:

$$\int \mathcal{D}g \int \mathcal{D}\phi \equiv \prod_\epsilon \int dl_\epsilon \, \mu(l_\epsilon) \prod_\sigma \int d\phi_\sigma$$

Choose the Hartle-Hawking wavefunctional [5] and the spacetime triangulation:

$$\Psi_{\text{HH}}[g,\phi] = \mathcal{N} \int \mathcal{D}G \int \mathcal{D}\Phi \; e^{iS[g,\phi,G,\Phi]}$$



Evaluate the trace of the square of reduced density matrix operator [6]:

$$\text{Tr}_M \, \hat{\rho}_M^2 = 0.977 \pm 0.002$$

RESULT !!!

Consequences

- In [7] Penrose argues that gravity-matter entanglement is at odds with (classical) spacetime, seen as a (four-dimensional) differentiable manifold. In light of this, our result could be seen as a quantitative indicator that in quantum gravity one cannot talk of "matter in a point of space", i.e., this result could be seen as a confirmation of a "spacetime as an emergent phenomenon".
- Thus, generic gravity-matter entanglement could be seen as a possible candidate for a criterion for a plausible theory of quantum gravity.
- Entanglement is in standard quantum mechanics a generic consequence of the interaction. This new entanglement can be regarded as a consequence of an effective interaction (such as the "exchange interactions", which are a consequence of quantum statistics). This additional "effective interaction" can potentially lead to corrections to the weak equivalence principle.

References

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