THE COSMOLOGICAL CONSTANT
in piecewise-linear models of quantum gravity

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TOPICS

- **Introduction to CC**
- **CC in quantum field theory**
- **CC in quantum gravity**
- **CC in Regge quantum gravity model**
- **Conclusions**

INTRODUCTION

In the Standard Model coupled to General Relativity there are three dimensionful parameters:

- the Planck length l_p, fixing the gravitational scale,
- the cosmological constant $\Lambda_{\rm eff}$, determining the cosmological scale, and
- the Higgs mass m_H, determining the electroweak scale.

Taking the ratios wrt. Planck length, we obtain:

$$c_\Lambda \equiv \Lambda_{\rm eff} l_p^2 \approx 10^{-122} \quad \leftarrow \text{ CC problem!}$$
$$c_H \equiv m_H^2 l_p^2 \approx 10^{-34} \quad \leftarrow \text{ Hierarchy problem!}$$

All other coupling constants in the SM are $\lesssim 1$, which makes c_Λ and c_H *unusually small*. An intuitive question to ask is:

WHY DOES THIS HAPPEN?

QFT IN CURVED SPACETIME

In the framework of quantum field theory in curved spacetime, the quantization of matter fields (keeping gravity classical) makes things even worse:

- calculate the expectation value of the stress-energy tensor at one-loop order,

$$\langle \hat{T}_{\mu\nu}(\phi)\rangle = T^{\text{classical}}_{\mu\nu}(\phi) + T^{\text{1-loop}}_{\mu\nu}(\phi),$$

- evaluate stress-energy for the ground state of matter fields, $\phi = 0$,

$$\begin{aligned} \langle \hat{T}_{\mu\nu}(\phi)\rangle\Big|_{\phi=0} &= \underbrace{T^{\text{classical}}_{\mu\nu}(0)}_{0} + T^{\text{1-loop}}_{\mu\nu}(0) = \Big[\text{textbook by Birrell\&Davies}\Big] = \\ &= \frac{a_1}{l_p^4} g_{\mu\nu} + \frac{a_2}{l_p^2}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + a_3\left(\ldots R^2 \ldots\right) + \ldots, \end{aligned}$$

  where $a_1, a_2, a_3, \ldots$ are dimensionless constants of $\mathcal{O}(1)$,

- substitute into Einstein equations and read off the 1-loop-renormalized value of the CC:

$$\Lambda_{\text{eff}} = \Lambda_{\text{bare}} + \Lambda_{\text{matt}}, \qquad \text{where} \qquad \Lambda_{\text{matt}} \equiv -\frac{8\pi a_1}{l_p^2}.$$

QFT IN CURVED SPACETIME

Why is this even worse:

$$\begin{array}{ccccc} \Lambda_{\rm eff} l_p^2 & = & \Lambda_{\rm bare} l_p^2 & - & 8\pi a_1 \\ \uparrow & & \uparrow & & \uparrow \\ \mathcal{O}(10^{-122}) & & \text{arbitrary} & & \mathcal{O}(1) \end{array}$$

QFT IN CURVED SPACETIME

Why is this even worse:

$$
\begin{array}{ccccc}
\Lambda_{\text{eff}} l_p^2 & = & \Lambda_b l_p^2 & - & 8\pi a_1 \\
\uparrow & & \uparrow & & \uparrow \\
\mathcal{O}(10^{-122}) & & \text{arbitrary} & & \mathcal{O}(1)
\end{array}
$$

In order to satisfy this equation, one needs to choose Λ_b to arrange for the cancellation of type:

$$
\begin{aligned}
(1.\cdots) - (1.\cdots) \; = \; & 0.0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000000000 \\
& 0000000000000000000001\cdots
\end{aligned}
$$

FINE TUNING !!!

QFT IN CURVED SPACETIME

Fundamental problems with the QFT approach:

- gravity should not be treated as classical

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QFT IN CURVED SPACETIME

Fundamental problems with the QFT approach:

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- gravity is not renormalizable

  → no RGE's can be formulated

  → no concept of "running" of the CC, no β function exists

- QFT is "effective" rather than fundamental

  → QFT is an approximation of some fundamental non-QFT model

  → the CC fine-tuning problem may be an artifact of the approximation

  → related: naturalness is under pressure (absence of SUSY at LHC)

WE NEED A THEORY OF QUANTUM GRAVITY !!!

CC IN QUANTUM GRAVITY

How to discuss the CC in the quantum gravity context:

- follow the recipe (from the review by Joe Polchinski) —
  - construct a QG model,
  - formulate the observable corresponding to CC in the QG model,
  - study its spectrum,

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  1. does the observed CC value ($\sim 10^{-122}$) belong to the spectrum of the CC-observable?
  2. why do we observe this particular point in the spectrum?

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Famously, Steven Weinberg applied this recipe to string theory:

- string theory + multiverse + anthropic principle $\quad \Rightarrow \quad \Lambda_{\text{eff}} \to 0$.

BUT THERE ARE ALTERNATIVES !!!

CC IN REGGE QUANTUM GRAVITY

Our main assumptions:

- **nature has a physical cutoff at the Planck scale**

  $\Rightarrow$ there exists a physical triangulation of spacetime,

- **Regge quantum gravity model**

  $\Rightarrow$ edge-lengths in the triangulation are fundamental degrees of freedom,

- **well-defined semiclassical limit**

  $\Rightarrow$ the limit $\hbar \to 0$ exists, and leads to some "effective" classical theory,

- **quantum fluctuations of matter fields do not gravitate**

  $\Rightarrow$ classical CC exactly cancels vacuum fluctuations.

Formulate the model, evaluate the spectrum of the CC observable, and compare it to experiment...

CC IN REGGE QUANTUM GRAVITY

Definition of the path integral for the Regge quantum gravity model:

$$Z = \int \mathcal{D}g \int \mathcal{D}\phi \; e^{iS[g,\phi]} \overset{\text{def}}{=} \prod_{\epsilon \in T(\mathcal{M})} \int_0^\infty dL_\epsilon \; \mu(L) \prod_{r \in T(\mathcal{M})} \int_{-\infty}^\infty d\phi_r \; e^{iS[L,\phi]}.$$

Calculation of the semiclassical limit, $l_p \to 0$, involves the following integrals:

$$\int_{-\infty}^\infty dx \, e^{-x^2/l_p^2} = l_p\sqrt{\pi} \quad \text{and} \quad \int_{-L}^\infty dx \, e^{-x^2/l_p^2} = l_p\sqrt{\pi}\left[1 - \frac{l_p}{2L\sqrt{\pi}}e^{-L^2/l_p^2} + \dots\right].$$

Having a well-defined classical limit requires the suppression of the nonanalytic terms, which can be achieved only with an exponential measure:

$$\mu(L) = \exp\left(-\frac{1}{8\pi l_p^2}\Lambda_{\text{meas}} V_4[L]\right), \qquad \text{where} \qquad 0 < \Lambda_{\text{meas}} \ll \frac{1}{l_p^2}.$$

CRUCIAL PROPERTY OF QG KINEMATICS !!!

CC IN REGGE QUANTUM GRAVITY

How to calculate CC in QG? The effective action equation in QFT:

$$e^{i\Gamma[\phi]} = \int \mathcal{D}\chi \exp\left[iS[\phi+\chi] - i\int d^4x \frac{\delta\Gamma[\phi]}{\delta\phi}\chi\right].$$

This can be generalized to QG in a straightforward manner:

$$e^{i\Gamma[L,\phi]} = \int \mathcal{D}l\ \mu(L+l) \int \mathcal{D}\chi \exp\left[iS[L+l,\phi+\chi] - i\int d^4x \left(\frac{\delta\Gamma}{\delta L}l + \frac{\delta\Gamma}{\delta\phi}\chi\right)\right].$$

The classical action has the form

$$S[L,\phi] = S_R[L] + S_M[L,\phi] + \frac{1}{8\pi l_p^2}\Lambda_b V_4[L],$$

and matter fields are in the ground state,

$$\phi = 0, \qquad \frac{\delta\Gamma}{\delta\phi} = 0.$$

Effective action equation reduces to:

$$e^{i\Gamma[L,0]} = \int \mathcal{D}l\ e^{iS_R[L+l] + \frac{i}{8\pi l_p^2}(\Lambda_{\text{bare}} + i\Lambda_{\text{meas}})V_4[L+l] - i\int \frac{\delta\Gamma}{\delta L}l} \int \mathcal{D}\chi\, e^{iS[L+l,\chi]}.$$

CC IN REGGE QUANTUM GRAVITY

The matter path integral can in principle be evaluated as

$$\int \mathcal{D}\chi \, e^{iS[L+l,\chi]} = \exp\left[i\frac{\Lambda_{\mathrm{matt}}}{8\pi l_p^2} V_4[L+l] + (\text{terms not proportional to } V_4) \right],$$

so we obtain the effective action equation:

$$e^{i\Gamma[L,0]} = \int \mathcal{D}l \exp\left[\frac{i}{8\pi l_p^2} \left(\Lambda_{\mathrm{bare}} + \Lambda_{\mathrm{matt}} + i\Lambda_{\mathrm{meas}}\right) V_4[L+l] + \right.$$
$$\left. + (\text{terms not proportional to } V_4) - i\int d^4x \, \frac{\delta \Gamma}{\delta L} l \right].$$

After taking the semiclassical limit and Wick rotating, we obtain

$$\Gamma[L,0] = \frac{1}{8\pi l_p^2} \Lambda_{\mathrm{eff}} V_4[L+l] + (\text{terms not proportional to } V_4) + \mathcal{O}(l_p^2),$$

where the effective CC is given as:

$$\Lambda_{\mathrm{eff}} \equiv \Lambda_{\mathrm{bare}} + \Lambda_{\mathrm{matt}} + \Lambda_{\mathrm{meas}}.$$

CC IN REGGE QUANTUM GRAVITY

The spectrum of the CC observable is continuous, and depends on two free parameters, $\Lambda_{\mathbf{bare}}$ and $\Lambda_{\mathbf{meas}}$:

$$\begin{array}{ccccccc} \Lambda_{\rm eff} l_p^2 & = & \Lambda_{\rm bare} l_p^2 & + & \Lambda_{\rm matt} l_p^2 & + & \Lambda_{\rm meas} l_p^2. \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \mathcal{O}(10^{-122}) & & \text{arbitrary} & & \mathcal{O}(1) & & 0 < \ldots \ll 1 \end{array}$$

We we implement the assumption that matter quantum fluctuations do not gravitate by choosing $\Lambda_{\mathbf{bare}}$ to exactly cancel the matter contribution $\Lambda_{\mathbf{matt}}$ (NO FINE TUNING !!!),

$$\Lambda_{\rm bare} + \Lambda_{\rm matt} \stackrel{\rm def}{=} 0, \qquad \Rightarrow \qquad \Lambda_{\rm eff} = \Lambda_{\rm meas},$$

and we test that the observed CC is compatible with the remaining free parameter $\Lambda_{\mathbf{meas}}$:

$$0 < \Lambda_{\rm meas} l_p^2 \ll 1 \quad \Rightarrow \quad 0 < \Lambda_{\rm eff} l_p^2 \ll 1 \quad \Rightarrow \quad 0 < 10^{-122} \ll 1.$$

NONZERO VALUE OF THE CC IS A PURE QUANTUM GRAVITY EFFECT !!!

CONCLUSIONS

- The framework of QFT in curved spacetime is **not suitable** for the discussion of the CC.
- The framework of full QG is suitable, and the analysis of CC is well defined.
- In Regge QG model, the smallness of the CC is a (weak) prediction.
- This prediction is **nonperturbative** (i.e. completely invisible in the perturbation theory), due to the presence of nonanalytic terms in the semiclassical limit.
- Nonzero CC is interpreted as a pure QG effect.
- The measured value of the CC fits well to the predicted spectrum: $0 < 10^{-122} \ll 1$.

- ... and no need to appeal to the multiverse and the anthropic principle... :-)

THANK YOU!