SPINCUBE MODEL OF QG AND CONNECTION TO CDT

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Loops15, Erlangen

CONCEPTS

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Nature has a physical cutoff at the scale γl_p***.***

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The simplest implementation of the principle:

Spacetime is a piecewise-linear manifold.

Things to note:

- PL-structure (a triangulation) is not a regulator, but a physical entity;
- UV-completion: inside a 4-simplex, spacetime is flat and matter fields are constant;
- finite number of degrees of freedom (in a finite volume);
- field theory reconstructed only as an approximation, like in fluid mechanics.

SPINCUBE MODEL OF QG

Classical theory — constrained $BFCG$ action:

$$S = \int \underbrace{B^{ab} \wedge R_{ab} + e^a \wedge \nabla \beta_a}_{\text{topological theory}} - \underbrace{\phi^{ab} \wedge \left(B_{ab} - \varepsilon_{abcd}\, e^c \wedge e^d \right)}_{\text{simplicity constraint}} .$$

Basic properties:

- equivalent to general relativity,
- similar in structure to the Plebanski action,
- contains tetrad fields in the topological sector,
- coupling to matter fields completely straightforward,
- based on $2BF$ action for the Poincaré 2-group.

SPINCUBE MODEL OF QG

Comparison of labels in spinfoam models and spincube model:

object	symbol	spinfoams	spincube
vertex	v		
edge	ϵ		$l \in \mathbb{R}_0^+$
triangle	Δ	$j \in \mathbb{N}_0/2$	$m \in \mathbb{Z}$
tetrahedron	τ	$\iota \in \mathbb{N}_0/2$	$M = 1$
4-simplex	σ		

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Quantum theory — the spincube state sum:

$$Z = \sum_{T \in \mathcal{T}} \int_{\mathbb{R}_0^+} dl_1 \dots \int_{\mathbb{R}_0^+} dl_E \sum_{m_1 \in \mathbb{Z}} \dots \sum_{m_F \in \mathbb{Z}} \prod_{\epsilon \in T} \mathcal{A}_\epsilon(l,m) \prod_{\Delta \in T} \mathcal{A}_\Delta(l,m) \prod_{\sigma \in T} e^{iS_\sigma(l,m)}.$$

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Edge and triangle amplitudes $\mathcal{A}_\epsilon$ and $\mathcal{A}_\Delta$ are chosen such that they impose the simplicity constraint between l's and m's:

$$|m_\Delta| = \frac{1}{\gamma l_p^2} A_H\left(l_{\epsilon_1}, l_{\epsilon_2}, l_{\epsilon_3}\right), \qquad \epsilon_1, \epsilon_2, \epsilon_3 \in \Delta, \qquad \forall \Delta \in T.$$

SIMPLICITY CONSTRAINT

Problem of imposing the simplicity constraint:

$$\underbrace{F}_{\substack{\text{number of}\\ \text{triangles}}} \geqslant \underbrace{E}_{\substack{\text{number of}\\ \text{edges}}} ,$$

so the system of equations for l's is overdetermined, leading to $F - E$ irrational Diophantine equations for m's.

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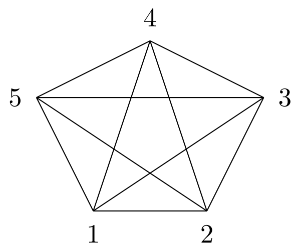
Either impose the simplicity constraint weakly (i.e. on-shell, see talk by A. Miković), or prove that the system of Diophantine equations has a nonempty set of solutions!

SIMPLICITY CONSTRAINT

The case of one 4-simplex ($E = F = 10$):



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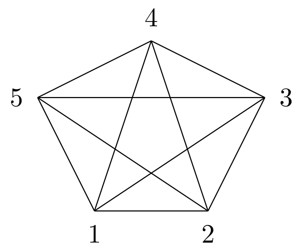

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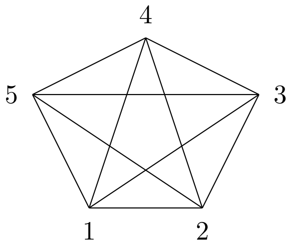

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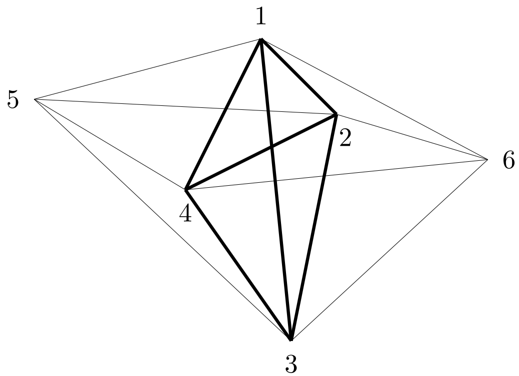
The case of two 4-simplices ($E = 14$, $F = 16$):



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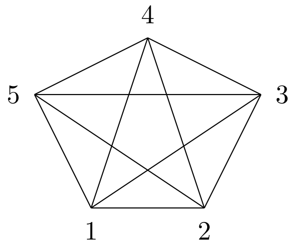

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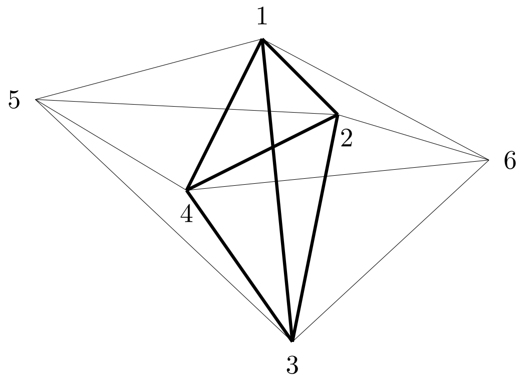

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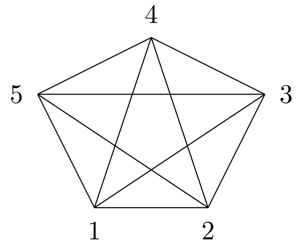

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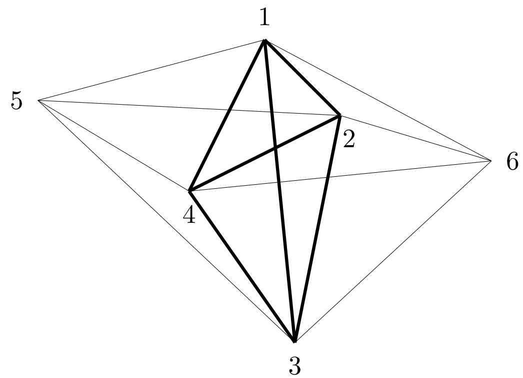

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The case of general triangulation ($F \geqslant E$): proof by induction! In the generic case, all 4-simplices in the triangulation must be equal.

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In general, given a solution of type

$$l_\epsilon = L_\epsilon(m_1, \ldots, m_{10}), \qquad \forall \epsilon \in T,$$

we implement the simplicity constraint by choosing the amplitudes as

$$\mathcal{A}_\epsilon(l,m) = \delta(l_\epsilon - L_\epsilon), \qquad \mathcal{A}_\Delta(l,m) = \chi\left(|m_\Delta| - \frac{1}{\gamma l_p^2} A_H(L_{\epsilon\in\Delta})\right),$$

so the spincube state sum becomes:

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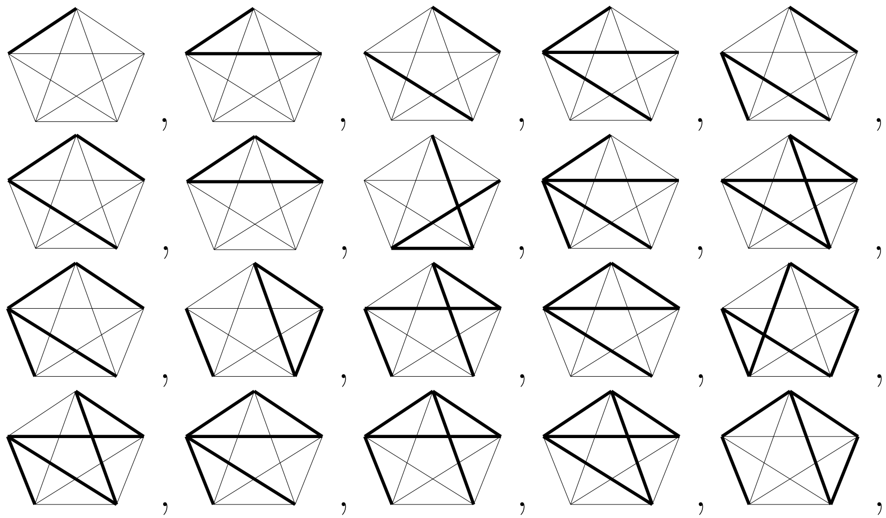
Since $m_1, \ldots, m_{10}$ belong to the same 4-simplex, we can commute the sums and write

$$Z = \sum_{m_1\in\mathbb{Z}} \cdots \sum_{m_{10}\in\mathbb{Z}} \left(\sum_{T\in\mathcal{T}} e^{iS_R(L(m))} \right).$$

RELATION TO CDT

Consider the special case of isosceles 4**-simplices, such that** $l_\epsilon \in \{a, b\}$**. There are** 40 **such simplices in total:**



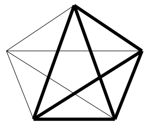
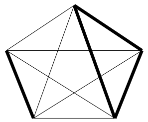
plus the "thick$\leftrightarrow$**thin" inverted ones ("thin"**$= a$**, "thick"**$= b$**). In general, these simplices contain four types of triangles,**

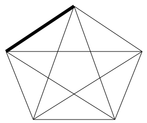
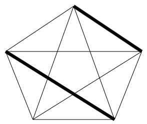
$$(a, a, a), \qquad (a, a, b), \qquad (a, b, b), \qquad (b, b, b),$$

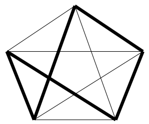
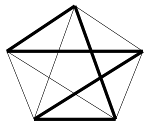
so the simplicity constraint consists of 4 **equations for** 2 **variables, and has no solution.**

RELATION TO CDT

But three pairs of simplices contain only two types of triangles,

 ,  , containing only (a, a, b) and (b, b, b),

 ,  , containing only (a, a, a) and (a, a, b),

 ,  , containing only (a, a, b) and (a, b, b).

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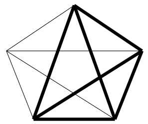
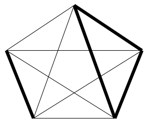
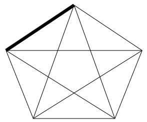
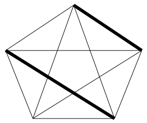
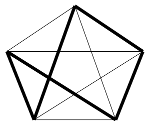
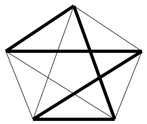
 ,  , containing only (a, a, b) and (b, b, b),

 ,  , containing only (a, a, a) and (a, a, b),

 ,  , containing only (a, a, b) and (a, b, b).

For these the simplicity constraint reduces to the system of two equations for two variables, and can always be solved. The state sum can then be written as:

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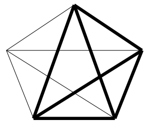
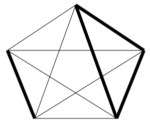
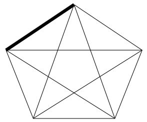
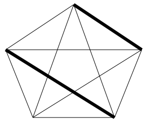
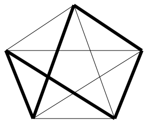
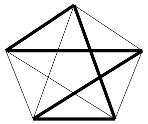
 ,  , containing only (a, a, b) and (b, b, b),

 ,  , containing only (a, a, a) and (a, a, b),

 ,  , containing only (a, a, b) and (a, b, b).

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CONCLUSIONS

Overview of results:

- simplicity constraint can be imposed strongly, so that the spincube model is well defined, despite overcomplete system of equations;
- two classes of solutions: identical irregular 4-simplices, and three types of isosceles 4-simplices;
- CDT is one of isosceles solutions, and thus a special case of the spincube model.

Topics for further research:

- classify solutions containing only $3, 4, \dots, 9$ different edges;
- repeat the CDT phase-transition analysis for all other classes, study what happens to phases when one varies the edge-length parameters;
- introduce matter fields, study realistic QG systems;
- study the relationship between the two semiclassical limits: average over many triangulations, or take $l, m \to \infty$, or both?

$\Rightarrow$ **More people is needed to do all this!**

THANK YOU!