HAMILTONIAN STRUCTURE OF THE BFCG THEORY

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THE PROBLEM OF QUANTUM GRAVITY

Why quantize gravity?

- same reasons as electrodynamics (two-slit experiment, hydrogen atom, ...)
- resolution of singularities (black holes, Big Bang, ...)
- black hole information paradox (nonunitary evolution?)
- theoretical and aesthetical reasons...

How to quantize gravity?

- perturbation theory does not work (nonrenormalizability of gravity)...
- almost zero experimental results to guide us...
- ...we have a problem!

LOOP QUANTUM GRAVITY

The idea

- Wilson loops are chosen as basic degrees of freedom,
- formalized as "spin network states",
- canonically quantized.

Achievements

- nonperturbative quantization of GR,
- kinematic sector of the theory well-defined,
- lengths, areas and volumes of space quantized!

Drawbacks

- dynamics described only in principle,
- no proof of semiclassical limit,
- very limited possibility for calculations.

SPINFOAM MODELS

The idea

- build up on canonical LQG (use the same degrees of freedom, construct the same structure of the Hilbert space, etc.),
- rewrite GR action using the Plebanski formalism,

$$S = \int B_{ab} \wedge R^{ab} + \phi^{abcd} B_{ab} \wedge B_{cd},$$

- discretize spacetime into 4-simplices,
- perform covariant quantization of the BF sector, by providing a definition for the gravitational path integral,

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \exp\left[i \sum_{\Delta} B_{\Delta} R_{\Delta} \right] = \ldots = \sum_{\Lambda} \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v),$$

- enforce the Plebanski constraint by restricting the representations Λ and redefining the vertex amplitude A_4.

SPINFOAM MODELS

Achievements

- well-defined nonperturbative quantum theory of gravity,
- both kinematical and dynamical sectors under control,
- can have a proper semiclassical limit.

Drawbacks

- geometry is "fuzzy" at the Planck scale,
- has many different semiclassical limits,
- matter coupling is problematic,
- hard to extract any results.

The reason for these drawbacks: tetrads are not explicitly present in the action!

THE $BFCG$ ACTION

One can associate the $BFCG$ action to the Poincaré 2-group:

$$S = \int B_{ab} \wedge R^{ab} + C_a \wedge G^a, \qquad (G^a = d\beta^a + \omega^a{}_b \wedge \beta^b).$$

Note that the Lagrange multiplier C^a is a 1-form and has an equation of motion $\nabla C^a = 0$, exactly the same as the tetrad e!

Therefore,

$$\left.\begin{array}{ll} \bullet\ \textbf{identify:} & C^a \equiv e^a, \\ \bullet\ \textbf{rename:} & BFCG \to BFEG, \end{array}\right\} \boxed{\textbf{KEY STEP}}$$

and rewrite the action as

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a.$$

THE CONSTRAINED $BFCG$ ACTION

The $BFCG$ action can be constrained to give GR:

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}} .$$

Equations of motion are equivalent to:

- equations that determine the multipliers and β:

$$\phi^{ab} = R^{ab}, \qquad B^{ab} = \varepsilon^{abcd} e_c \wedge e_d, \qquad \beta^a = 0$$

- Einstein equations:

$$\varepsilon_{abcd} R^{bc} \wedge e^d = 0,$$

- no-torsion equation:

$$\nabla e^a = 0.$$

This is classically equivalent to general relativity!

THE MAIN BENEFITS

Introduction of matter fields is straightforward:

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a - \phi_{ab} \left(B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right) +$$

$$+ i\kappa \int \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \bar{\psi} \left(\gamma^d \overleftrightarrow{d} + \{\omega, \gamma^d\} + \frac{im}{2}\, e^d \right) \psi -$$

$$- i \frac{3\kappa}{4} \int \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c\, \bar{\psi} \gamma_5 \gamma^d \psi, \qquad (\kappa = \frac{8}{3} \pi l_p^2).$$

The covariant quantization is possible — spincube model:

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \int \mathcal{D}e \int \mathcal{D}\beta \exp \left[i \sum_{\Delta} B_{\Delta} R_{\Delta} + \sum_{l} e_l G_l \right] = \ldots =$$

$$= \sum_{\Lambda} \prod_{p} A_1(\Lambda_p) \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v).$$

THE HAMILTONIAN STRUCTURE

The $BFCG$ action in components:

$$S = \int d^4x\, \varepsilon^{\mu\nu\rho\sigma} \left[B_{ab\mu\nu} \left(\partial_\rho \omega^{ab}{}_\sigma + \omega^a{}_{c\rho} \omega^{cb}{}_\sigma \right) + e_{a\mu} \left(\partial_\nu \beta^a{}_{\rho\sigma} + \omega^a{}_{c\nu} \beta^c{}_{\rho\sigma} \right) \right] .$$

The variables:

$$B^{ab}{}_{\mu\nu}(x), \qquad e^a{}_\mu(x), \qquad \omega^{ab}{}_\mu(x) \quad \text{and} \quad \beta^a{}_{\mu\nu}(x).$$

Momenta and primary constraints:

$$\begin{array}{rclrcl} P(B)_{ab}{}^{\mu\nu} & \equiv & \pi(B)_{ab}{}^{\mu\nu} \approx 0, & P(e)_a{}^\mu & \equiv & \pi(e)_a{}^\mu \approx 0, \\ P(\omega)_{ab}{}^0 & \equiv & \pi(\omega)_{ab}{}^0 \approx 0, & P(\omega)_{ab}{}^i & \equiv & \pi(\omega)_{ab}{}^i - 2\varepsilon^{0ijk} B_{abjk} \approx 0, \\ P(\beta)_a{}^{0i} & \equiv & \pi(\beta)_a{}^{0i} \approx 0, & P(\beta)_a{}^{ij} & \equiv & \pi(\beta)_a{}^{ij} + 2\varepsilon^{0ijk} e_{ak} \approx 0. \end{array}$$

The simultaneous Poisson brackets:

$$\begin{array}{rcl} \{ B^{ab}{}_{\mu\nu}(\vec{x},t) \,,\, \pi(B)_{cd}{}^{\rho\sigma}(\vec{x}',t) \} & = & 4\delta^a_{[c}\delta^b_{d]}\delta^\rho_{[\mu}\delta^\sigma_{\nu]}\delta^{(3)}(\vec{x}-\vec{x}'), \\ \{ e^a{}_\mu(\vec{x},t) \,,\, \pi(e)_b{}^\nu(\vec{x}',t) \} & = & \delta^a_b \delta^\nu_\mu \delta^{(3)}(\vec{x}-\vec{x}'), \\ \{ \omega^{ab}{}_\mu(\vec{x},t) \,,\, \pi(\omega)_{cd}{}^\nu(\vec{x}',t) \} & = & 2\delta^a_{[c}\delta^b_{d]}\delta^\nu_\mu \delta^{(3)}(\vec{x}-\vec{x}'), \\ \{ \beta^a{}_{\mu\nu}(\vec{x},t) \,,\, \pi(\beta)_b{}^{\rho\sigma}(\vec{x}',t) \} & = & 2\delta^a_b \delta^\rho_{[\mu}\delta^\sigma_{\nu]}\delta^{(3)}(\vec{x}-\vec{x}'). \end{array}$$

THE HAMILTONIAN STRUCTURE

The canonical Hamiltonian:

$$H_c = \int d^3\vec{x}\,\varepsilon^{0ijk}\left[-B_{ab0i}R^{ab}{}_{jk} - e^a{}_0 G_{aijk} - 2\beta_{a0k}T^a{}_{ij} - \omega_{ab0}\left(\nabla_i B^{ab}{}_{jk} - e^a{}_i\beta^b{}_{jk}\right)\right],$$

The total Hamiltonian:

$$H_T = H_c + \int d^3\vec{x}\left[\lambda(B)^{ab}{}_{\mu\nu}P(B)_{ab}{}^{\mu\nu} + \lambda(e)^a{}_\mu P(e)_a{}^\mu + \right.$$

$$\left. +\lambda(\omega)^{ab}{}_\mu P(\omega)_{ab}{}^\mu + \lambda(\beta)^a{}_{\mu\nu}P(\beta)_a{}^{\mu\nu}\right].$$

Consistency of the primary constraints:

$$\begin{aligned} \dot{P}(B)_{ab}{}^{0i} &= 2\varepsilon^{0ijk}S(R)_{abjk}, \\ \dot{P}(e)_a{}^0 &= S(G)_a, \\ \dot{P}(\beta)_a{}^{0i} &= 2\varepsilon^{0ijk}S(T)_{ajk}, \\ \dot{P}(\omega)_{ab}{}^0 &= 2S(Be\beta)_{ab}, \end{aligned} \qquad \text{where} \qquad \begin{aligned} S(R)^{ab}{}_{jk} &\equiv R^{ab}{}_{jk} \approx 0, \\ S(G)^a &\equiv \varepsilon^{0ijk}G^a{}_{ijk} \approx 0, \\ S(T)^a{}_{ij} &\equiv T^a{}_{ij} \approx 0, \\ S(Be\beta)^{ab} &\equiv \varepsilon^{0ijk}\left[\nabla_i B^{ab}{}_{jk} - e^{[a}{}_i\beta^{b]}{}_{jk}\right] \approx 0. \end{aligned}$$

THE HAMILTONIAN STRUCTURE

Determined multipliers:

$$\begin{array}{lcl} \dot{P}(B)_{ab}{}^{jk} \approx 0 & & \lambda(\omega)^{ab}{}_i = \frac{1}{2}\nabla_i \omega^{ab}{}_0, \\ \dot{P}(e)_a{}^k \approx 0 & & \lambda(\beta)^a{}_{ij} = \nabla_{[i}\beta^a{}_{0j]} - \frac{1}{2}\omega^a{}_{b0}\beta^b{}_{ij}, \\ & \text{implies} & \\ \dot{P}(\beta)_a{}^{jk} \approx 0 & & \lambda(e)^a{}_i = \nabla_i e^a{}_0 - \omega^a{}_{b0}e^b{}_i, \\ \dot{P}(\omega)_{ab}{}^k \approx 0 & & \lambda(B)^{ab}{}_{ij} = \dfrac{1}{2}\left(\nabla_{[i}B^{ab}{}_{0j]} + \omega^{[a}{}_{c0}B^{b]c}{}_{ij}\right) + \\ & & \qquad + \dfrac{1}{4}\left(e^{[a}{}_0\beta^{b]}{}_{ij} + e^{[a}{}_j\beta^{b]}{}_{0i} - e^{[a}{}_i\beta^{b]}{}_{0j}\right). \end{array}$$

Consistency of secondary constraints is automatic:

$$\begin{aligned} \dot{S}(R)^{ab}{}_{ij} &= 2\omega^{[a}{}_{c0}S(R)^{b]c}{}_{ij}, \\ \dot{S}(G)^a &= \varepsilon^{0ijk}\beta_{b0k}S(R)^{ab}{}_{ij} - \omega^a{}_{b0}S(G)^b, \\ \dot{S}(T)^a{}_{ij} &= \frac{1}{2}e_{b0}S(R)^{ab}{}_{ij} - \omega^a{}_{b0}S(T)^b{}_{ij}, \\ \dot{S}(Be\beta)^{ab} &= 2\varepsilon^{0ijk}\left(B^{[a}{}_{c0k}S(R)^{b]c}{}_{ij} + \beta^{[a}{}_{0k}S(T)^{b]}{}_{ij}\right) + e^{[a}{}_0 S(G)^{b]} + 2\omega^{[a}{}_{c0}S(Be\beta)^{b]c}. \end{aligned}$$

THE HAMILTONIAN STRUCTURE

Algebra of constraints:

$$
\begin{aligned}
\{\, P(B)_{ab}{}^{jk} \,,\, P(\omega)_{cd}{}^{i} \,\} &= 8\varepsilon^{0ijk}\eta_{a[c}\eta_{bd]}\delta^{(3)}, \\
\{\, P(e)_{a}{}^{k} \,,\, P(\beta)_{b}{}^{ij} \,\} &= -2\varepsilon^{0ijk}\eta_{ab}\delta^{(3)}, \\
\{\, S(G)^{a} \,,\, P(\beta)_{b}{}^{jk} \,\} &= 2\varepsilon^{0ijk}\left[\delta^a_b\partial_i\delta^{(3)} + \omega^a{}_{bi}\delta^{(3)}\right], \\
\{\, S(G)^{a} \,,\, P(\omega)_{cd}{}^{i} \,\} &= 2\varepsilon^{0ijk}\delta^a_{[c}\beta_{d]jk}\delta^{(3)}, \\
\{\, S(T)^{a}{}_{ij} \,,\, P(e)_{b}{}^{k} \,\} &= \delta^a_b\partial_{[i}\delta^k_{j]}\delta^{(3)} + \omega^a{}_{b[i}\delta^k_{j]}\delta^{(3)}, \\
\{\, S(T)^{a}{}_{ij} \,,\, P(\omega)_{cd}{}^{k} \,\} &= \left(\delta^a_{[c}e_{d]j}\delta^k_i - \delta^a_{[c}e_{d]i}\delta^k_j\right)\delta^{(3)}, \\
\{\, S(Be\beta)^{ab} \,,\, P(e)_{c}{}^{i} \,\} &= -\varepsilon^{0ijk}\delta^{[a}_c\beta^{b]}{}_{jk}\delta^{(3)}, \\
\{\, S(Be\beta)^{ab} \,,\, P(\beta)_{c}{}^{jk} \,\} &= -2\varepsilon^{0ijk}e^{[a}{}_i\delta^{b]}_c\delta^{(3)}, \\
\{\, S(Be\beta)^{ab} \,,\, P(\omega)_{cd}{}^{i} \,\} &= 2\varepsilon^{0ijk}\left(\delta^a_{[c}B_{d]}{}^b{}_{jk} + \delta^b_{[c}B^a{}_{d]jk}\right)\delta^{(3)}, \\
\{\, S(Be\beta)^{ab} \,,\, P(B)_{cd}{}^{jk} \,\} &= 4\varepsilon^{0ijk}\left[\delta^a_{[c}\delta^b_{d]}\partial_i\delta^{(3)} + \left(\omega^a{}_{[ci}\delta^b_{d]} + \delta^a_{[c}\omega^b{}_{d]i}\right)\delta^{(3)}\right], \\
\{\, S(R)^{ab}{}_{ij} \,,\, P(\omega)_{cd}{}^{k} \,\} &= 2\delta^a_{[c}\delta^b_{d]}\left(\delta^k_j\partial_i\delta^{(3)} - \delta^k_i\partial_j\delta^{(3)}\right) + \\
&\quad +2\left(\delta^a_{[c}\omega_{d]}{}^b{}_j\delta^k_i - \delta^a_{[c}\omega_{d]}{}^b{}_i\delta^k_j + \omega^a{}_{[ci}\delta^b_{d]}\delta^k_j - \omega^a{}_{[cj}\delta^b_{d]}\delta^k_i\right)\delta^{(3)}.
\end{aligned}
$$

THE HAMILTONIAN STRUCTURE

First class constraints:

$$P(B)_{ab}{}^{0i}, \qquad P(e)_a{}^0, \qquad P(\omega)_{ab}{}^0, \qquad P(\beta)_a{}^{0i},$$

Second class constraints:

$$P(B)_{ab}{}^{jk}, \qquad P(e)_a{}^i, \qquad P(\omega)_{ab}{}^i, \qquad P(\beta)_a{}^{ij},$$

$$S(R)^{ab}{}_{ij}, \qquad S(G)^a, \qquad S(Be\beta)^{ab}, \qquad S(T)^a{}_{ij}.$$

THE HAMILTONIAN STRUCTURE

The gauge symmetry generator:

$$G[\varepsilon^{ab}{}_i, \varepsilon^{ab}, \varepsilon^a, \varepsilon^a{}_i] = \int d^3\vec{x} \left[\frac{1}{2} \left(\dot{\varepsilon}^{ab}{}_i P(B)_{ab}{}^{0i} - \varepsilon^{ab}{}_i \mathcal{G}_{ab}{}^i \right) + \left(\dot{\varepsilon}^a P(e)_a{}^0 - \varepsilon^a \mathcal{G}_a \right) + \right.$$

$$\left. + \frac{1}{2} \left(\dot{\varepsilon}^{ab} P(\omega)_{ab}{}^0 - \varepsilon^{ab} \mathcal{G}_{ab} \right) + \left(\dot{\varepsilon}^a{}_i P(\beta)_a{}^{0i} - \varepsilon^a{}_i \mathcal{G}_a{}^i \right) \right],$$

where

$$\mathcal{G}_{ab}{}^i \equiv 2\varepsilon^{0ijk} S(R)_{abjk} + \nabla_j P(B)_{ab}{}^{ji} + 2\omega^c{}_{[a0} P(B)_{b]c}{}^{0i},$$

$$\mathcal{G}_{ab} \equiv 2S(Be\beta)_{ab} + \nabla_i P(\omega)_{ab}{}^i + 2\omega^c{}_{[a0} P(\omega)_{b]c}{}^0 - 2e_{[a0} P(e)_{b]}{}^0 - 2e_{[ai} P(e)_{b]}{}^i +$$

$$+ B_{c[aij} P(B)_{b]}{}^{cij} + 2B_{c[a0i} P(B)_{b]}{}^{c0i} - 2\beta_{[a0i} P(\beta)_{b]}{}^{0i} - \beta_{[aij} P(\beta)_{b]}{}^{ij},$$

$$\mathcal{G}_a \equiv S(G)_a + \nabla_i P(e)_a{}^i - \omega^b{}_{a0} P(e)_b{}^0 - \frac{1}{2}\beta^b{}_{0i} P(B)_{ab}{}^{0i} - \frac{1}{4}\beta^b{}_{ij} P(B)_{ab}{}^{ij},$$

$$\mathcal{G}_a{}^i \equiv 2\varepsilon^{0ijk} S(T)_{ajk} + \nabla_j P(\beta)_a{}^{ji} - \omega^b{}_{a0} P(\beta)_b{}^{0i} - \frac{1}{2} e^b{}_0 P(B)_{ab}{}^{0i} + \frac{1}{2} e^b{}_j P(B)_{ab}{}^{ij}.$$

THE HAMILTONIAN STRUCTURE

Form-variations of the variables:

$$
\begin{aligned}
\{\, \omega^{ab}{}_{\mu}\,, G \,\} &= \nabla_{\mu}\varepsilon^{ab}, \\
\{\, B^{ab}{}_{\mu\nu}\,, G \,\} &= 2\nabla_{[\mu}\varepsilon^{ab}{}_{\nu]} - \varepsilon^{a}{}_{c}B^{cb}{}_{\mu\nu} - \varepsilon^{b}{}_{c}B^{ac}{}_{\mu\nu} + \varepsilon^{[a}\beta^{b]}{}_{\mu\nu} - 2\varepsilon^{[a}{}_{[\mu}e^{b]}{}_{\nu]}, \\
\{\, \beta^{a}{}_{\mu\nu}\,, G \,\} &= 2\nabla_{[\mu}\varepsilon^{a}{}_{\nu]} - \varepsilon^{a}{}_{b}\beta^{b}{}_{\mu\nu}, \\
\{\, e^{a}{}_{\mu}\,, G \,\} &= \nabla_{\mu}\varepsilon^{a} - \varepsilon^{a}{}_{b}e^{b}{}_{\mu}. \qquad \left(\varepsilon^{ab}{}_{0} \equiv 0, \varepsilon^{a}{}_{0} \equiv 0\right)
\end{aligned}
$$

Symmetry transformation corresponding to $\varepsilon^{ab}(x)$:

$$\omega'_{\mu} = \Lambda\omega_{\mu}\Lambda^{-1} + \Lambda\partial_{\mu}\Lambda^{-1}, \quad e' = \Lambda e, \quad \beta' = \Lambda\beta, \quad B' = \Lambda B\Lambda^{T}, \quad \Lambda \in SO(3,1).$$

Symmetry transformation corresponding to $\varepsilon^{ab}{}_{i}(x)$:

$$B'^{ab}{}_{\mu\nu} = B^{ab}{}_{\mu\nu} + 2\nabla_{[\mu}\varepsilon^{ab}{}_{\nu]}(x), \qquad e' = e, \qquad \omega' = \omega, \qquad \beta' = \beta.$$

Symmetry transformation corresponding to $\varepsilon^{a}{}_{i}(x)$:

$$\beta'^{a}{}_{\mu\nu} = \beta^{a}{}_{\mu\nu} + 2\nabla_{[\mu}\varepsilon^{a}{}_{\nu]}, \qquad B'^{ab}{}_{\mu\nu} = B^{ab}{}_{\mu\nu} - 2e^{[a}{}_{[\mu}\varepsilon^{b]}{}_{\nu]}, \qquad e' = e, \qquad \omega' = \omega.$$

Symmetry transformation corresponding to $\varepsilon^{a}(x)$:

$$e'^{a}{}_{\mu} = e^{a}{}_{\mu} + \nabla_{\mu}\varepsilon^{a}, \qquad B'^{ab}{}_{\mu\nu} = B^{ab}{}_{\mu\nu} + \varepsilon^{[a}\beta^{b]}{}_{\mu\nu}, \qquad \beta' = \beta, \qquad \omega' = \omega.$$

THE HAMILTONIAN STRUCTURE

Introduce new set of parameters:

$$\begin{aligned} \varepsilon^a &\rightarrow \xi^\lambda e^a{}_\lambda, & \varepsilon^a{}_\mu &\rightarrow \varepsilon^a{}_\mu + \xi^\lambda \beta^a{}_{\lambda\mu}, \\ \varepsilon^{ab} &\rightarrow \varepsilon^{ab} + \xi^\lambda \omega^{ab}{}_\lambda, & \varepsilon^{ab}{}_\mu &\rightarrow \varepsilon^{ab}{}_\mu + \xi^\lambda B^{ab}{}_{\lambda\mu}. \end{aligned}$$

The generator in terms of new parameters:

$$\begin{aligned} G[\varepsilon^{ab}{}_i, \varepsilon^a{}_i, \varepsilon^{ab}, \xi^\lambda] = \int d^3\vec{x} \left[\frac{1}{2} \left(\dot{\varepsilon}^{ab}{}_i P(B)_{ab}{}^{0i} - \varepsilon^{ab}{}_i \mathcal{G}_{ab}{}^i \right) + \left(\dot{\varepsilon}^a{}_i P(\beta)_a{}^{0i} - \varepsilon^a{}_i \mathcal{G}_a{}^i \right) + \right. \\ \left. + \frac{1}{2} \left(\dot{\varepsilon}^{ab} P(\omega)_{ab}{}^0 - \varepsilon^{ab} \mathcal{M}_{ab} \right) + \left(\dot{\xi}^\lambda \Pi_\lambda + \xi^0 \mathcal{P}_0 + \xi^i \mathcal{P}_i \right) \right], \end{aligned}$$

where

$$\begin{aligned} \Pi_\lambda &= \frac{1}{2} B^{ab}_{\lambda i} P(B)_{ab}{}^{0i} + \frac{1}{2} \omega^{ab}{}_\lambda P(\omega)_{ab}{}^0 + \beta^a{}_{\lambda i} P(\beta)_a{}^{0i} + e^a{}_\lambda P(e)_a{}^0, \\ \mathcal{M}_{ab} &= \mathcal{G}_{ab}, \\ \mathcal{P}_0 &= \mathcal{H}_T, \\ \mathcal{P}_i &= \ldots \end{aligned}$$

CANONICAL QUANTIZATION

Fields $\phi^A \in \{B^{ab}{}_{\mu\nu}, \beta^a{}_{\mu\nu}, \omega^{ab}{}_\mu, e^a{}_\mu\}$ **and their momenta** π_A **are promoted to operators,**

$$\phi^A \to \hat{\phi}^A = \phi^A, \qquad \pi_A \to \hat{\pi}_A = i\frac{\delta}{\delta\phi^A},$$

the wavefunctional $\Psi[\phi^A] \equiv \langle\phi^A|\Psi\rangle$ **is required to be gauge-invariant,**

$$\hat{G}\,\Psi[\phi^A] = 0,$$

and the set of solutions of this equation determines the physical Hilbert space of the theory:

$$\mathcal{H}_{\rm Phys} = \{\ \Psi[\phi^A] \quad | \quad \hat{G}\,\Psi[\phi^A] = 0\ \}.$$

TODO: repeat the whole calculation for the constrained $BFCG$ **model!**

THANK YOU!