# SPINCUBE MODEL OF QUANTUM GRAVITY

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# THE PROBLEM OF QUANTUM GRAVITY

## Why quantize gravity?

- same reasons as electrodynamics (two-slit experiment, hydrogen atom, . . . )
- resolution of singularities (black holes, Big Bang, . . . )
- black hole information paradox (nonunitary evolution??!!)
- theoretical and aesthetical reasons. . .

## How to quantize gravity?

- perturbation theory does not work (nonrenormalizability of gravity). . .
- almost zero experimental results to guide us. . .
- . . . we have a problem!

# NONRENORMALIZABILITY OF GRAVITY

**Perturbative quantization idea**

- expand the metric around flat spacetime,

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x),$$

- use it to expand the Einstein-Hilbert action,

$$S_{EH} = \int d^4x\, \sqrt{-g}R = \int d^4x\, h_{\mu\nu}\Box h^{\mu\nu} + h^3 + h^4 + \ldots,$$

- obtain a theory for self-interacting massless spin-2 field in flat spacetime,
- quantize in analogy to Yang-Mills theories.

# NONRENORMALIZABILITY OF GRAVITY

**However, the resulting theory is nonrenormalizable:**

- tree-level Feynman diagrams are finite,
- one-loop diagrams require a counterterm — remove it by renormalizing $g_{\mu\nu}$,
- two-loop diagrams require a counterterm of type

$$c_2 \frac{1}{\varepsilon^2} R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}, \qquad (\varepsilon \to 0)$$

  which **cannot be removed by renormalization**.

- each higher-loop diagram requires another nonrenormalizable counterterm.

**The theory contains infinitely many coupling constants!**

**The theory loses its predictive power — each choice of coupling constants fixes a different theory of gravity!**

# PROBLEM OF QUANTUM GRAVITY

$$S = S_{EH} + \int d^4x \, c_2 \mathcal{L}_2 + c_3 \mathcal{L}_3 + \ldots$$

**The problem of quantizing gravity "reduces" to**

$\gg$ **inventing a SET OF FIRST PRINCIPLES that Nature obeys** $\ll$

**such that all coupling constants** $c_1, c_2, \ldots$ **are fixed and can be calculated.**

**Mainstream candidate approaches:**

- String theory
- Noncommutative geometry
- Loop quantum gravity
- Causal dynamical triangulations
- Causal set theory
- Doubly special relativity

. . . and so on. . .

**There is no experimental data at the Planck scale, to distinguish between these ideas.**

# LOOP QUANTUM GRAVITY

## The idea of LQG

- Wilson loops are chosen as basic degrees of freedom,
- formalized as "spin network states",
- canonically quantized.

## Achievemenets

- nonperturbative quantization of GR,
- kinematic sector of the theory well-defined,
- lengths, areas and volumes of space quantized!

## Drawbacks

- dynamics described only in principle,
- no proof of semiclassical limit,
- very limited possibility for calculations.

# SPINFOAM MODELS

## The idea in brief

- build up on canonical LQG (use the same degrees of freedom, construct the same structure of the Hilbert space, etc.),
- discretize spacetime into 4-simplices,
- perform covariant quantization, by providing a definition for the gravitational path integral,
$$Z = \int \mathcal{D}g_{\mu\nu} \exp\left(iS_{EH}[g_{\mu\nu}]\right),$$
- use this definition to calculate expectation values for all interesting observables as in quantum field theory.

# SPINFOAM MODELS

## The idea in a bit more detail

- rewrite GR action using Plebanski formalism:

$$S = \int B_{ab} \wedge R^{ab} + \text{“Plebanski constraint”},$$

- quantize the $BF$ sector by (a) triangulating the spacetime manifold, (b) defining the path integral

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \exp\left[ i \sum_{\Delta} B_{\Delta} R_{\Delta} \right] = \ldots = \sum_{\Lambda} \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v),$$

  where $\Lambda$ are irreducible representations of $SO(3,1)$, while $A_2$ and $A_4$ are chosen such that $Z$ is a topological invariant — the resulting theory is a TQFT (in the sense of Atiyah);

- enforce the “Plebanski constraint” by projecting the representations from $SO(3,1)$ to $SU(2)$, and by redefining the vertex amplitude $A_4$,

- obtain a non-topological path integral definition of the theory, with local degrees of freedom.

# SPINFOAM MODELS

**Main achievements**

- well-defined nonperturbative quantum theory of gravity,
- both kinematical and dynamical sectors under control,
- can be made to have a proper semiclassical limit,
- predicts the values of the counterterm coupling constants.

**Main drawbacks**

- geometry is “fuzzy” at Planck scale,
- distances between spacetime points not well-defined,
- matter coupling is problematic,
- hard to extract any results.

**The reason for these drawbacks: tetrads are not explicitly present in the action!**

# POINCARÉ GROUP

**Properties of the Poincaré group:**

- $P(4) = \mathbb{R}^4 \ltimes SO(3,1)$
- Lorentz group has a connection 1-form $\omega$ which transforms as a gauge potential

$$\omega \to g^{-1}\omega g + g^{-1}dg, \qquad (g : \mathcal{M}_4 \to SO(3,1))$$

- one can introduce line holonomies

$$g_l(\omega) = \exp \int_l \omega$$

- 4-translation group has a tetrad 1-form $e$ which **does not** transform as a gauge potential! ($\mathbb{I} - e$ does)
- one can associate a $BF$ action to the Lorentz group,

$$S = \int B_{ab} \wedge R^{ab}, \qquad (R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb})$$

while the translation group is ignored!

# 2-CATEGORIES AND 2-GROUPS

**Category theory**

- a category is a structure with "objects" and "morphisms",
- a group is a category with only one object and invertible morphisms.

**2-category theory**

- a 2-category is a structure with "objects", "morphisms" and "2-morphisms",
- a 2-group is a category with only one object and invertible morphisms and 2-morphisms.

**Crossed module** $(G, H, \triangleright, \partial)$

- $G$ and $H$ are Lie groups,
- $\triangleright$ is an action of $G$ on $H$ ($\triangleright : G \times H \to H$),
- $\partial$ is a homomorphism of $H$ on $G$ ($\partial : H \to G$).

**Theorem: every 2-group is isomorphic to an appropriate crossed module**

# POINCARÉ 2-GROUP

**Properties of the Poincaré 2-group:**

- $(G, H, \triangleright, \partial)$, where:

  $$G = SO(3,1), \qquad H = \mathbb{R}^4, \qquad \triangleright : SO(3,1) \times \mathbb{R}^4 \to \mathbb{R}^4 \qquad \partial : \mathbb{R}^4 \to SO(3,1)$$

- Lorentz group has a connection 1-form $\omega$, but the 2-Poincaré structure generates in addition a 2-form $\beta$, such that $(\omega, \beta)$ is called a 2-connection, and transforms as

  $$\omega \to g^{-1}\omega g + g^{-1}dg, \qquad \beta \to g^{-1} \triangleright \beta, \qquad (g : \mathcal{M}_4 \to SO(3,1))$$
  $$\omega \to \omega + \underbrace{\partial\eta}_{0}, \qquad \beta \to \beta + d\eta + \omega \wedge^{\triangleright} \eta + \underbrace{\eta \wedge \eta}_{0}, \qquad (\eta : \mathcal{M}_4 \to \mathbb{R}^4)$$

- one can introduce line holonomies and surface holonomies

  $$g_l(\omega) = \exp \int_l \omega, \qquad h_f(\beta) = \exp \int_f \beta,$$

- one can associate the $BFCG$ (also called $2BF$) action to the Poincaré 2-group:

  $$S = \int B_{ab} \wedge R^{ab} + C_a \wedge G^a, \qquad (G^a = d\beta^a + \omega^a{}_b \wedge \beta^b).$$

# TETRAD FIELDS IN THE $BFCG$ ACTION

**Note that the Lagrange multiplier $C^a$**

- is a 1-form,
- transforms as

$$C \to g^{-1} \triangleright C, \qquad C \to C \quad \text{wrt. } \eta \text{ transformations,}$$

- has an equation of motion $\nabla C^a = 0$.

**The multiplier $C$ has exactly the same properties as the tetrad $e$!**

**Therefore, make an identification**

$$C^a \equiv e^a,$$

**and rewrite the $BFCG$ action as**

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a$$

**This action is topological, and the 2-group structure enables us to perform the spinfoam-like quantization.**

# NEW ACTION FOR GENERAL RELATIVITY

**The $BFCG$ action can be constrained to give GR:**

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left( B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}} \; .$$

**Equations of motion are:**

- $\delta\phi :$ $\qquad B^{ab} - \varepsilon^{abcd} e_c \wedge e_d = 0,$
- $\delta\beta :$ $\qquad \nabla e^a = 0,$
- $\delta\omega :$ $\qquad \nabla B^{ab} - e^{[a} \wedge \beta^{b]} = 0,$
- $\delta B :$ $\qquad R^{ab} - \phi^{ab} = 0,$
- $\delta e :$ $\qquad \nabla \beta_a + 2\varepsilon_{abcd} \phi^{bc} \wedge e^d = 0,$

# NEW ACTION FOR GENERAL RELATIVITY

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**Equations of motion are (after some cleaning-up...):**

- equations that determine the multipliers and $\beta$:

$$\phi^{ab} = R^{ab}, \qquad B^{ab} = \varepsilon^{abcd} e_c \wedge e_d, \qquad \beta^a = 0$$

- Einstein equations:

$$\varepsilon_{abcd} R^{bc} \wedge e^d = 0,$$

- no-torsion equation:

$$\nabla e^a = 0.$$

**This is classically equivalent to general relativity!**

# THE SPINCUBE MODEL

**The spincube quantization procedure:**

- rewrite GR action as a topological theory plus constraint:

$$S = \int \underbrace{B_{ab} \wedge R^{ab} + e^a \wedge G_a}_{\text{topological sector}} - \underbrace{\phi_{ab} \left( B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right)}_{\text{constraint}},$$

- quantize the $BFCG$ sector by (a) triangulating the spacetime manifold, (b) defining the path integral

$$Z = \int \mathcal{D}\omega \int \mathcal{D}B \int \mathcal{D}e \int \mathcal{D}\beta \exp \left[ i \sum_{\Delta} B_{\Delta} R_{\Delta} + \sum_{l} e_l G_l \right] = \ldots =$$

$$= \sum_{\Lambda} \prod_{p} A_1(\Lambda_p) \prod_{f} A_2(\Lambda_f) \prod_{v} A_4(\Lambda_v),$$

where $\Lambda$ are irreducible 2-representations of Poincaré 2-group, while $A_1$, $A_2$ and $A_4$ are chosen such that $Z$ is a topological invariant (a 2-TQFT),

# THE SPINCUBE MODEL

**The spincube quantization procedure:**

- enforce the constraint $B^{ab} = \varepsilon^{abcd} e_c \wedge e_d$ by projecting representations $\Lambda$ to a subset that satisfies the Heron formula for the area of a triangle,

$$|m_f| l_p^2 = A(\Delta) \equiv \sqrt{s(s-l_1)(s-l_2)(s-l_3)}, \qquad (s = \frac{l_1 + l_2 + l_3}{2}),$$

- redefine the vertex amplitudes $A_1$, $A_2$ and $A_4$ so that the theory is finite and has a correct classical limit,

- obtain a non-topological path integral definition of the theory, with local degrees of freedom.

**Main achievements:**

- geometry is Regge-like at Planck scale,

- distances between spacetime points are well-defined,

- matter coupling is straightforward,

- easier to calculate with.

# MATTER FIELDS

**Introduction of matter fields is straightforward:**

- at the classical level the fermionic matter can be added to the action due to the explicit presence of the tetrads in the topological sector:

$$S = \int B_{ab} \wedge R^{ab} + e^a \wedge G_a - \phi_{ab} \left( B^{ab} - \varepsilon^{abcd} e_a \wedge e_b \right) +$$
$$+ i\kappa \int \varepsilon_{abcd} \, e^a \wedge e^b \wedge e^c \wedge \bar{\psi} \left( \gamma^d \overset{\leftrightarrow}{d} + \{\omega, \gamma^d\} + \frac{im}{2} e^d \right) \psi -$$
$$- i\frac{3\kappa}{4} \int \varepsilon_{abcd} e^a \wedge e^b \wedge \beta^c \, \bar{\psi} \gamma_5 \gamma^d \psi, \qquad (\kappa = \frac{8}{3}\pi l_p).$$

- scalar fields, Yang-Mills fields, Immirzi parameter, cosmological constant, ..., can be added in a similar manner,

- performing the spincube quantization with the new action amounts to introducing additional labels and terms in the vertex amplitude $A_4$ which describe the matter degrees of freedom and their coupling to gravity.

# APPLICATIONS OF SPINCUBE MODEL

**How can all this be useful in any sense?**

- spincube quantization provides one with a concrete quantum theory of gravity with matter,
- one can calculate the effective action using the discretization of the QFT formula

$$e^{i\Gamma(\phi)} = \int \mathcal{D}\varphi \exp\left[ iS[\phi+\varphi] - i\int d^4x \frac{\partial \Gamma}{\partial \phi}\varphi \right],$$

**When matter fields are present in the model, quantum corrections in the effective action can enable one to discuss:**

- resolution of the black hole and cosmological singularities,
- detailed analysis of the black hole information paradox,
- renormalization properties of QFT,
- deep Planck-scale regime of space, time and matter,
- motivation for further fundamental questions…

***THANK YOU!***