FINITENESS AND EFFECTIVE ACTION OF THE MODIFIED ELPR/FK SPIN FOAM MODEL

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arXiv:1101.3294, arXiv:1104.1384

THE PROBLEM

The ELPR/FK state sum is given as:

$$Z_\sigma = \sum_j \int d\vec{n} \prod_{f\in\sigma} A_{\mathrm{face}}(j) \prod_{e\in\sigma} A_{\mathrm{edge}}(j,\vec{n}) \prod_{v\in\sigma} A_{\mathrm{vertex}}(j,\vec{n}),$$

where

$$A_{\mathrm{face}}(j) = 2j+1, \qquad A_{\mathrm{edge}} = 1, \qquad A_{\mathrm{vertex}}(j,\vec{n}) = W_v^{ELPR/FK}(j,\vec{n}).$$

Simple power counting suggests that Z_σ diverges (and also $Z = \sum_\sigma Z_\sigma$).

Finiteness of Z_σ (and Z) is still an OPEN PROBLEM!

The strategy: redefine the vertex amplitude A_{vertex} to make the state sum finite.

This was successfully done for the Barrett-Crane model (Crane, Perez, Rovelli, 2001) and also suggested for the ELPR/FK model (Perini, Rovelli, Speziale, 2009).

THE PROPOSED SOLUTION

Redefine the vertex amplitude as:

$$A_{\text{vertex}}(j,\vec{n}) = \frac{1}{\prod_{f\in v}(2j_f+1)^p} W_v^{ELPR/FK}(j,\vec{n}),$$

where p is a parameter to be tuned.

Proof of finiteness:

$$|Z_\sigma| \leqslant \sum_j \int d\vec{n}\ \prod_f (2j_f+1) \prod_v \frac{1}{\prod_{f\in v}(2j_f+1)^p} |W_v^{ELPR/FK}(j,\vec{n})|.$$

Since $|W_v^{ELPR/FK}(j,\vec{n})| \leqslant const$, we have $\int d\vec{n} = const$, and:

$$|Z_\sigma| \leqslant const \prod_f \sum_{j_f} \frac{1}{(2j_f+1)^{N_f p-1}},$$

where N_f is the number of vertices bounding the face f.

The sum over j_f will converge if $N_f p - 1 > 1$. Since $N_f \geqslant 2$, a sufficient condition for absolute convergence is

$$p > 1.$$

The choice of p is INDEPENDENT OF THE TWO-COMPLEX σ!

Consequently, $|Z_\sigma| < \infty$, and so also Z_σ is finite, for all σ.

Furthermore, using the "summation = refining" argument (Rovelli, Smerlak, 2010), we have

$$Z = \sum_\sigma Z_\sigma = Z_{\cup\sigma} < \infty.$$

But, how do we determine the actual value of p?

THE EFFECTIVE ACTION

The large-spin asymptotics of the ELPR/FK vertex amplitude is (Barrett et al., 2009)

$$W_v^{ELPR/FK} = a(v)e^{i\gamma S_R(v)} + b(v)e^{-i\gamma S_R(v)},$$

so introduce another redefinition of the vertex amplitude (Miković, Vojinović, 2010):

$$A_{\text{vertex}} = \frac{1}{\prod_{f\in v}(2j_f+1)^p}\frac{W_v + \sqrt{W_v^2 - 4ab}}{2a} \qquad \Big(\approx \frac{1}{\prod_{f\in v}(2j_f+1)^p} e^{i\gamma S_R(v)} \Big).$$

Then one can employ the background-field method to calculate an effective action in the large-spin limit,

$$e^{i\Gamma(j,\vec{n})} = \sum_{j'} \int d\vec{n}' \prod_f A_{\text{face}}(j+j') \prod_v A_{\text{vertex}}(j+j', \vec{n}+\vec{n}'),$$

and obtain the result:

$$\Gamma \approx \frac{1}{16\pi l_p^2} S_{EH} + \sum_f c_f(p)\log j_f + \frac{1}{2}\operatorname{tr}\log S_R'' + O(j^{-1}).$$

The coefficients $c_f(p)$ can be determined experimentally, and they determine the value of p!

CONCLUSIONS

If we redefine the spin foam vertex amplitude as

$$A_{\text{vertex}} = \frac{1}{\displaystyle\prod_{f\in v}(2j_f+1)^p}\frac{W_v+\sqrt{W_v^2-4ab}}{2a},$$

we obtain a spin foam model with the following properties:

- **it keeps all good properties of the original ELPR/FK model,**
- **it is finite by construction,**
- **it has general relativity as a classical limit,**
- **first-order correction terms can be calculated,**
- **diffeomorphism invariance is recovered in the continuum limit,**
- **the parameter p can be determined experimentally by measuring the first-order corrections to the classical action.**

THANK YOU!