EFFECTIVE ACTION AND FINITENESS OF THE MODIFIED ELPR/FK SPINFOAM MODEL

Aleksandar Miković (Lusofona University and GFM – University of Lisbon)
Marko Vojinović (GFM – University of Lisbon)

arXiv:1101.3294, arXiv:1104.1384

INTRODUCTION TO SPINFOAMS

Spinfoam formalism is a way to formulate a theory of quantum gravity, motivated by the framework of LQG.

A spinfoam model is constructed in the following way:

- **discretize the spacetime into 4-simplices (like in Regge calculus),**
- **construct a 2-complex σ dual to the triangulation,**
- **assign labels l and amplitudes $A(l)$ to vertices, edges, faces, etc. of the 2-complex,**
- **define the state sum (a discretized path-integral) as:**

$$Z_\sigma = \sum_l \prod_{v\in\sigma} A_{\text{vertex}}(l) \prod_{e\in\sigma} A_{\text{edge}}(l) \prod_{f\in\sigma} A_{\text{face}}(l) \dots$$

Optionally, one can sum over all possible 2-complexes σ ("third quantization"):

$$Z = \sum_\sigma \omega_\sigma Z_\sigma \,.$$

The resulting model represents a *covariant*, *nonperturbative* and *background-independent* quantum theory of gravity.

INTRODUCTION TO SPINFOAMS (cont.)

The labels l and amplitudes $A(l)$ are chosen so that the following requirements are met:

- **the boundary Hilbert space of the model should coincide with the Hilbert space constructed in the canonical quantization framework,**
- **the model should be finite,**
- **the model should reduce to general relativity in the classical limit.**

Most famous spinfoam models:

- **Ponzano-Regge and Turaev-Viro in 3D,**
- **Ooguri, Crane-Yetter, Barret-Crane in 4D,**
- **Engle-Pereira-Rovelli-Livine / Freidel-Krasnov (ELPR/FK).**

All these models partially satisfy the above requriements.

THE FINITENESS PROBLEM

The ELPR/FK state sum is given as:

$$Z_\sigma = \sum_j \int d\vec{n} \prod_{f\in\sigma} A_{\text{face}}(j) \prod_{e\in\sigma} A_{\text{edge}}(j,\vec{n}) \prod_{v\in\sigma} A_{\text{vertex}}(j,\vec{n}),$$

where

$$A_{\text{face}}(j) = 2j+1, \qquad A_{\text{edge}} = 1, \qquad A_{\text{vertex}}(j,\vec{n}) = W_v^{ELPR/FK}(j,\vec{n}).$$

Simple power counting suggests that Z_σ diverges (and also $Z = \sum_\sigma Z_\sigma$).

Finiteness of Z_σ (and Z) is still an OPEN PROBLEM!

The strategy: redefine the vertex amplitude A_{vertex} to make the state sum finite.

This was successfully done for the Barrett-Crane model (Crane, Perez, Rovelli, 2001) and also suggested for the ELPR/FK model (Perini, Rovelli, Speziale, 2009).

THE FINITENESS PROBLEM (cont.)

Redefine the vertex amplitude as:

$$A_{\text{vertex}}(j,\vec{n}) = \frac{1}{\prod_{f\in v}(2j_f+1)^p} W_v^{ELPR/FK}(j,\vec{n}),$$

where p is a parameter to be tuned.

Proof of finiteness:

$$|Z_\sigma| \leqslant \sum_j \int d\vec{n}\ \prod_f (2j_f+1) \prod_v \frac{1}{\prod_{f\in v}(2j_f+1)^p} |W_v^{ELPR/FK}(j,\vec{n})|.$$

Since $|W_v^{ELPR/FK}(j,\vec{n})| \leqslant const$, we have $\int d\vec{n} = const$, and:

$$|Z_\sigma| \leqslant const \prod_f \sum_{j_f} \frac{1}{(2j_f+1)^{N_f p-1}},$$

where N_f is the number of vertices bounding the face f.

<u>THE FINITENESS PROBLEM</u> (cont.)

The sum over j_f will converge if $N_f p - 1 > 1$. Since $N_f \geqslant 2$, a sufficient condition for absolute convergence is

$$p > 1.$$

The choice of p is INDEPENDENT OF THE TWO-COMPLEX σ!

Consequently, $|Z_\sigma| < \infty$, and so also Z_σ is finite, for all σ.

Furthermore, using the "summation = refining" argument (Rovelli, Smerlak, 2010), we have

$$Z = \sum_\sigma \omega_\sigma Z_\sigma = Z_{\cup\sigma} < \infty.$$

Question: how do we determine the actual value of p?

THE CLASSICAL LIMIT PROBLEM

The classical regime is actually the large-spin regime of a spinfoam model.

The large-spin asymptotics of the ELPR/FK vertex amplitude is (Barrett et al., 2009)

$$W_v^{ELPR/FK} = a(v)e^{i\gamma S_R(v)} + b(v)e^{-i\gamma S_R(v)}.$$

Then one can employ the background field method to calculate an effective action in the large-spin limit,

$$e^{i\Gamma(j,\vec{n})} = \sum_{j'} \int d\vec{n}' \, \prod_f A_{\text{face}}(j+j') \prod_v A_{\text{vertex}}(j+j', \vec{n}+\vec{n}'),$$

and obtain the result of type:

$$\Gamma \approx \log\cos S_R + \text{corrections}.$$

This result for the effective action is called the COSINE PROBLEM!

THE CLASSICAL LIMIT PROBLEM (cont.)

The strategy: introduce another redefinition of the vertex amplitude (Miković, MV, 2010):

$$A_{\mathrm{vertex}} = \frac{1}{\prod_{f\in v}(2j_f+1)^p}\frac{W_v+\sqrt{W_v^2-4ab}}{2a} \qquad \left(\approx \frac{1}{\prod_{f\in v}(2j_f+1)^p} e^{i\gamma S_R(v)}\right).$$

Calculate the effective action with the modified vertex amplitude and obtain the result:

$$\Gamma \approx \frac{1}{16\pi l_p^2} S_{EH} + \frac{1}{2}\,\mathrm{tr}\log S_R'' + \sum_f c_f(p)\log j_f + O(j^{-1}).$$

The effective action has the following structure:

- **the LO term is general relativity,**
- **there is the NLO term of the usual trace-log structure,**
- **there is an additional NLO term containing the parameter p.**

The value of the parameter p can be determined experimentally, through coefficients $c_f(p)$.

CONCLUSIONS

If we redefine the spin foam vertex amplitude as

$$A_{\text{vertex}} = \frac{1}{\prod_{f \in v} (2j_f + 1)^p} \frac{W_v + \sqrt{W_v^2 - 4ab}}{2a},$$

we obtain a spin foam model with the following properties:

- it keeps all good properties of the original ELPR/FK model,
- it is finite by construction,
- it has general relativity as a classical limit,
- first-order correction terms can be calculated,
- diffeomorphism invariance and triangulation independence are recovered in the continuum limit,
- the parameter p can be determined experimentally by measuring the first-order corrections to the classical action.

FURTHER RESEARCH

Prospects for future studies:

- **examine the effective action in more detail,**
- **study the triangulation (in)dependence of NLO terms,**
- **introduce matter fields and cosmological constant,**
- **apply the theory to cosmology, black holes, astrophysics, . . . ,**
- **ideas for GUTs, etc. . .**

THANK YOU!