Santa Claus as a Macroscopic Quantum Phenomenon

Marko Vojinović

Institute of Physics,
University of Belgrade, 2004.

Contents

1 Introduction **2**

2 Classical treatment **2**

3 Quantum treatment **3**

4 Shortcomings of the quantum treatment **4**

5 Conclusions **5**

Abstract

You have all probably already heard about the analysis of the Santa Claus problem in physics. Simply put, elementary calculation of the properties that Santa Claus should have in order to deliver presents under the Christmas tree of every well-behaved child during the Christmas eve necessarily leads to conclusion that such a task is impossible to do, when one sees the numerical values of properties involved. Nevertheless, modern quantum physics actually resolves this problem, and gives a satisfactory description of Santa Claus, as we shall outline in this paper.

1 Introduction

The first step in solving any problem is its proper formulation. Therefore, we first need to specify the question which is to be addressed and assumptions that we start with.

Santa Claus has a mission to deliver a present for every well-behaved child in the world, by putting it under the appropriate tree during one specific night, just before Christmas. Namely, he carries presents in a bag on his back, drives to each house in his flying sleigh pulled by flying reindeer, lands on the roof, climbs down the chimney, leaves the presents under the nicely decorated Christmas tree, climbs back through the chimney back to the roof and flies of to the next house. In buildings and houses with no chimney this scenario is slightly different, but this is not relevant for our analysis.

As we are interested only in gross structure, let us first solve following exercises, within given approximations (which can later be refined, if necessary):

1. Calculate the total number of well-behaved children in the world. Assume that well-behaved children constitute one quarter of the total population on the planet, which is roughly 6 billion.

2. Calculate the total number of houses that Santa should visit, as well as average distance between two houses with well-behaved children. Assume that the children are distributed homogeneously over continents spanning one quarter of total Earth surface, and that there is on average two children in each house. The Earth radius is 6 thousand kilometers.

3. Calculate total time available to Santa to deliver all the presents, as well as average time for each delivery. Consider the Christmas Eve to begin at $21:00$ hours on December 24th, and end at $7:00$ on December 25th. For simplicity, consider the problem nonrelativistically and with a very crude approximation that all children live in the same time zone. Then calculate the average velocity of the flying sleigh.

4. Estimate the total mass and volume of the bag with the presents which is to be carried by Santa. Imagine that each present takes up a box of one liter volume and weighs 200 grams, and consider that each child gets one such present.

Now that we have a well defined and well formulated problem, as well as the assumptions we start out with, let us turn to solving these simple exercises. Generally speaking, there are two possible treatments: classical and quantum.

2 Classical treatment

By simple and straightforward calculation we obtain the following results: the total number of well-behaved children in the world is 1.5 billion, and they live in 750 million houses. Using the radius of Earth, we find the area of its surface to be approximately 400 million square kilometers, while continents take up a quarter of this, ie. 100 million. Assuming that the houses are homogeneously distributed over this area, we obtain that the distance between two nearest-neighbor houses is 365 meters. This is (on average) the distance Santa should travel between two deliveries. With the assumptions stated above, total time that available for the whole mission is 10 hours, or 0.048 milliseconds per each house. The average velocity of the sleigh should be $2.7 \cdot 10^7 \, \frac{\mathrm{km}}{\mathrm{h}}$. Total volume of the bag with presents (assuming most tightly packed configuration of the presents) decreases with time, and at initial moment is $1.5 \cdot 10^6 \, \mathrm{m}^3$ (say, 600 of typical ten-floor home buildings). The initial mass of the bag is 300 thousand tons.

These results are discouraging, to say the least. Santa has no chance of performing his mission. Of course, we can try to refine our assumptions. For example, by taking into account different time zones where children live, Santa will have somewhat more time to deliver presents, which reduces the average velocity of the sleigh. Or, if Santa limits distribution of presents only to the Christian children which have Christmas trees, their number will be somewhat smaller. Finally, if we take into account that houses are not homogeneously distributed across the globe, but are clustered in cities and villages, the total distance

Santa must travel will be less. However, even with all these refinements, the numbers we would calculate would still be enormous. Alternatively, one could devise a scenario where Santa does not work alone, but rather has some logistic support like DHL service, but such models also have problems, and are not in accord with the spirit of the original idea of a Santa that works alone with his reindeer team.

All in all, this is a well known, often quoted result, from which we conclude that classical mechanics is not capable of giving a satisfactory description of Santa Claus. Nevertheless, having in mind the latest results that we shall present in the following, it would be completely wrong to deduce from this data that Santa cannot exist. On the contrary, we could say that classical mechanics (being essentially an approximate theory) does not work in this case, ie. is not good enough to describe Santa.

3 Quantum treatment

We now go on to consider one simple quantum model, in order to try and describe Santa with more success.

The estimate of number of children, houses, average distance and total travel time that Santa has available are no different than in the classical case, and the numerical results are the same. However, time available for one house, mean velocity of the sleigh, and total volume and mass of the bag with presents are now drastically different from the classical results. Let us discuss this in more detail.

The main paradigm (a “paradigm” is a philosophical starting point, a collection of concepts one uses to think about something) of quantum description of a physical system is based in the properties of the state that the system can be found in. One of those properties is the so-called superposition principle of states, which says that given a system which can be in state Ψ or in state Φ, it can also be in a superposition of these states, $\Psi + \Phi$. More precisely, a sum of states is also a state. For example, this property is being used when one wishes to describe the phenomenon of one particle passing through two openings (slits) simultaneously. If the state “particle goes through left slit” is denoted Ψ, while state “particle goes through right slit” is denoted Φ, then quantum mechanics claims that $\Psi + \Phi$, ie. the situation “particle goes both through left and right slits”, is also a valid possible state for the particle to be in (by the way, this is not just a theory, there are experiments verifying that these type of things really do happen). We then say that the particle is in the state of coherent superposition. The principle of superposition of the states is a fundamental property of nature, and it is very important.

The second paradigm of quantum description of a physical system is the influence of measurement to the system itself. In classical physics it was assumed that we can measure (ie. observe) some property of a physical system, without disturbing it in the process. In quantum physics this is not possible, and every measurement necessarily has some influence on the system, which typically might change the state of the system. This phenomenon is called the collapse of the state wavefunction, and can be nicely demonstrated on the example of the particle and two slits. Namely, if our particle is in the state $\Psi + \Phi$ (going through both slits simultaneously), we as experimenters have three distinct possibilities:

1. to measure whether the particle went through the left slit,
2. to measure whether the particle went through the right slit, or
3. not to measure the particle trajectory.

For example, if we choose the first possibility, and as a result we get that the particle indeed did go through the left slit, then we know that the particle did not go through the right slit. The state of the particle changed as a consequence of our measurement — the system was in state $\Psi + \Phi$ before the measurement, while after it is in state Ψ only. We say that the wavefunction “collapsed” from state $\Psi + \Phi$ to Ψ. Should have we measured that the particle did not go through the left slit, we would have known that it did go through the right one, ie. the wavefunction would collapse from $\Psi + \Phi$ into Φ.

Similar thing happens if we choose the second possibility. However, if we choose not to measure the particle trajectory, then there is no collapse, the state of the particle stays $\Psi + \Phi$, and we conclude that it went through both slits simultaneously.

The process of wavefunction collapse is also called decoherence, because the system goes from a state of coherent superposition into a state which is not a coherent superposition, ie. a decoherence of the state happens.

Now that we have mastered two basic features of quantum mechanics, we can go back to the Santa Claus problem and try to describe him from the quantum standpoint. But there we are in for a big surprise!

Consider Santa in a quantum state $\Psi(k)$ delivering a present into a k-th house (number $k \in \mathbb{N}$ counts all houses Santa should visit, and takes values from 1 to $7.5 \cdot 10^8$). He carries in a bag only two presents which he will leave in the house. Let us sum now all states $\Psi(k)$ for all possible values of k. The result is, by the superposition principle, also a possible state for the Santa, a coherent superposition of all single-house states. If Santa is in this coherent superposition state, then he actually leaves presents in all houses simultaneously (much like the particle going through two slits simultaneously). But this means that he has full ten hours to drive the sleigh to the house, with just two presents in the bag, hide and wait where no one would see him, leave the presents below the tree, and drive off after that! And this in turn means that there is no need for high-velocity sleigh, no 300 thousand tons of presents, no need for only 0.048 milliseconds of time available for delivery!

And that is not all, quantum mechanics provides us with a natural explanation why no child has ever seen Santa! Simply, if some child would see him, his position would be measured, which would lead to collapse of his wavefunction, and this would mean that he did not leave presents in any other house, which is not correct, since all well-behaved children always find his presents below their Christmas tree in the morning. So, if your child once asks you "Why do I never get to see Santa?", the correct answer is: "This is so other children could get their presents, too."

As we can see, quantum mechanics gives a very realistic and satisfactory description of Santa Claus, and in addition gives new predictions, ie. answers questions which classical mechanics was unable to address. Just as a particle can go through two slits simultaneously, so can Santa leave presents in all 750 million houses simultaneously. Moreover, in order to prevent decoherence of Santa's quantum superposition state, no child must ever see him leave presents, which is exactly what actually happens every Christmas eve. We conclude, as we promised at the end of previous section, that quantum mechanics gives successful description of Santa Claus, which is consistent with his existence, and only shows that classical mechanics is unable to account for proper description.

4 Shortcomings of the quantum treatment

Everyone who wants to do serious science (physics beforemost), must be aware of the shortcomings of any model or theory, no matter how successful it may seem. Critique is necessary — without it there is no progress in science, because in that case there would be no new questions raised and answered.

So, what problems might arise in our quantum model of Santa Claus? There are two main issues:

1. Since Santa is in the state of coherent superposition while he leaves presents to the children, these presents are also in the state of coherent superposition. Actually, it is just one single present which, like Santa himself, is to be found under all Christmas trees simultaneously. So when tomorrow morning one child finds the present and opens it, there should be a decoherence of the present, and it should disappear from under all other trees. But this does not happen!

2. How come only Santa, and no one else, can be in the coherent superposition state, meaning in several places at once? Why cannot everyone else do the same trick at their convenience?

These questions are rather serious, and may undermine our model, if they are not addressed appropriately. The answers, of course, exist, and we shall now evaluate to what extent they are appropriate and sufficient. Let us deal with them in turn.

First of all, if we apply the laws of quantum mechanics consistently, it is quite correct that the first child, by opening the present, is going to perform a measurement over it, which will result in the present's wavefuction collapsing and disappearing from below all other well-behaved children's Christmas trees. Remember, this was precisely the reason why no child was allowed to see Santa in the first place. There is no resolution to this problem, and the only remedy is to think of a different, little more elaborate model of Santa Claus. This is of course easy enough. For example, assume that Santa has a big hangar somewhere on the North Pole, filled with a billion and a half presents. On Christmas eve, he gets into a coherent superposition of new states, $\Psi(k,j)$, where every state $\Psi(k,j)$ describes Santa leaving j-th present in the k-th house! This means that Santa is in a superposition of states where he is leaving a different present in each house. Tomorrow morning, when a well-behaved child looks under a tree, it will find a present, and there will indeed be a collapse of the present's wavefunction, but this time it will disappear from the hangar on the North Pole, not from under other children's trees. Therefore every well-behaved child can get its own present, and the problem is avoided. Furthermore, different children can get different presents, which is also confirmed by real world observation. By the way, this is an entirely possible scenario, and it actually happens in the phenomenon called quantum teleportation (a subject of intense research in recent years).

As for the second question, why really is only Santa able to be in a coherent superposition state? Well, first, the phenomenon in which a physical system of large, macroscopic scale exhibits quantum behavior, like Santa in our case, is called "macroscopic quantum phenomenon" (look at the title of this text!). Second, it is not true that only Santa Claus is able to do it. There are several such phenomena, and we shall name three of them:

- the phenomenon called "superconductivity", which describes flow of electric current through some materials with zero resistance,
- the phenomenon called "superfluidity", which describes the flow of certain fluids (like liquid helium, for example) completely without friction and viscosity, and
- the phenomenon called "ferromagnetism", which describes the existence of permanent magnets (typically made of iron, for example).

Yes, if you did not know, the magnets that we all have in our houses and like to play with cannot be described with classical, but only with quantum mechanics!

In other words, macroscopic quantum phenomena are maybe rare to find, but nevertheless do exist, so why not Santa Claus as well?

5 Conclusions

Let us summarize. We have demonstrated, by explicit construction of the model, that the phenomenon of Santa Claus cannot be described within the framework of classical mechanics, but can be (with a little effort) successfully described within the quantum theory. Therefore, if you want to receive a present from the Santa Claus in the upcoming Christmas eve, you must behave well, decorate your Christmas tree, and study quantum mechanics!

There is only one question that we did not address above: how come that Santa's sleigh and reindeer can fly? If you wish to become successful physicists one day, try to make a model of Santa Claus which will be able to account for this. In the meantime, sometimes we all need to dream a little, right? ;-)